

65
NATIONAL BUREAU OF STANDARDS REPORT

7386a

FIGURES FOR
BLACK BOX MAXIMIZATION
of

CIRCULAR COVERAGE

by

C.T. Zahn, Jr.

Applied Mathematics Division

To

U.S. Army Signal Air Defense Engineering Agency



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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Operations Research Section

Applied Mathematics Division

Final Report (Volume Ia) to the

U.S. Army Signal Air Defense Engineering Agency

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ABSTRACT

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51	" (6,1/2)
52	" (6,1/2)
53	" (10,3/8)
54	" (10,3/8)
55	" (10,3/8)
56	" (10,3/8)
57	" (9,5/16)
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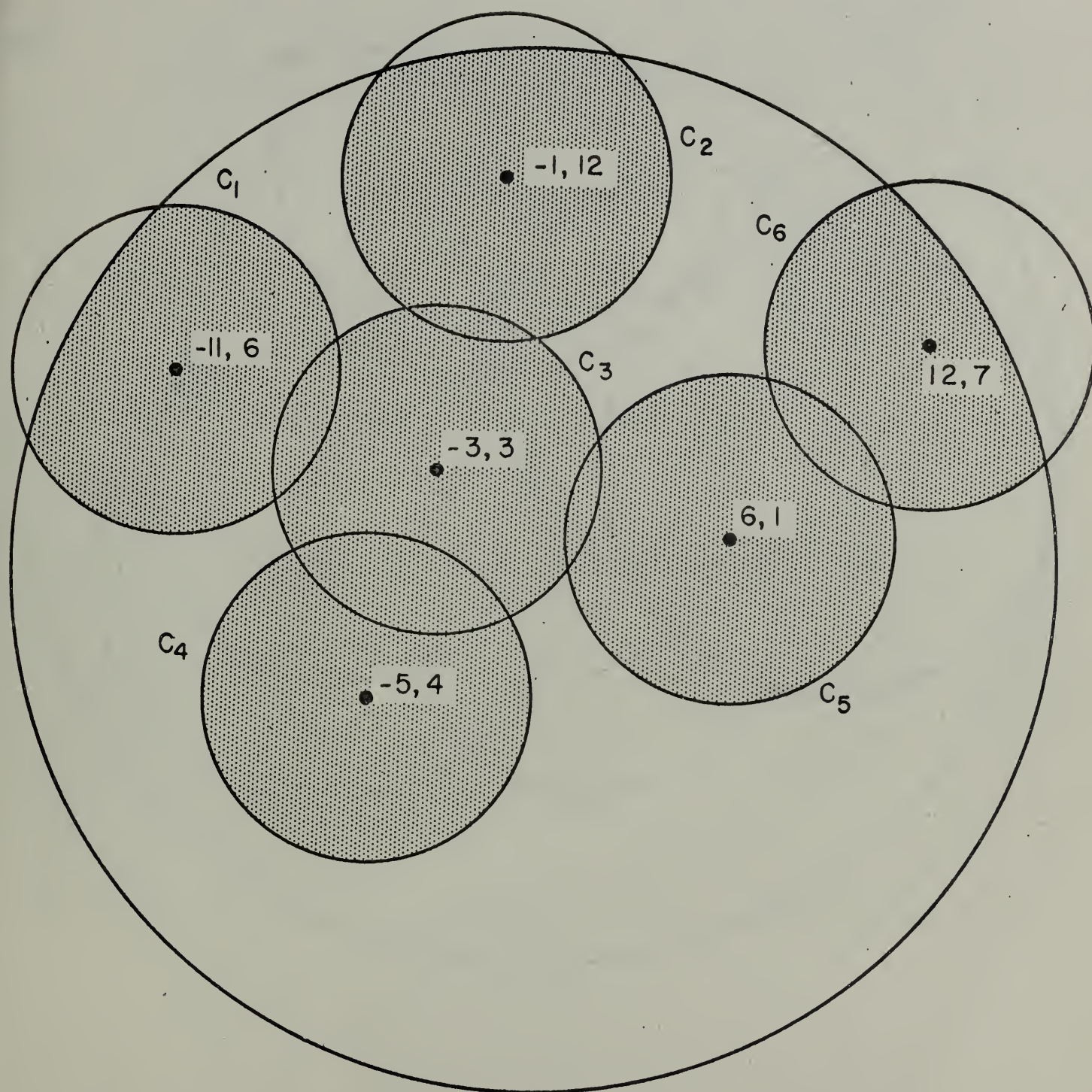


FIG. 1 COVERAGE BY SIX DISCS

$(3, \frac{7}{16})$
.574

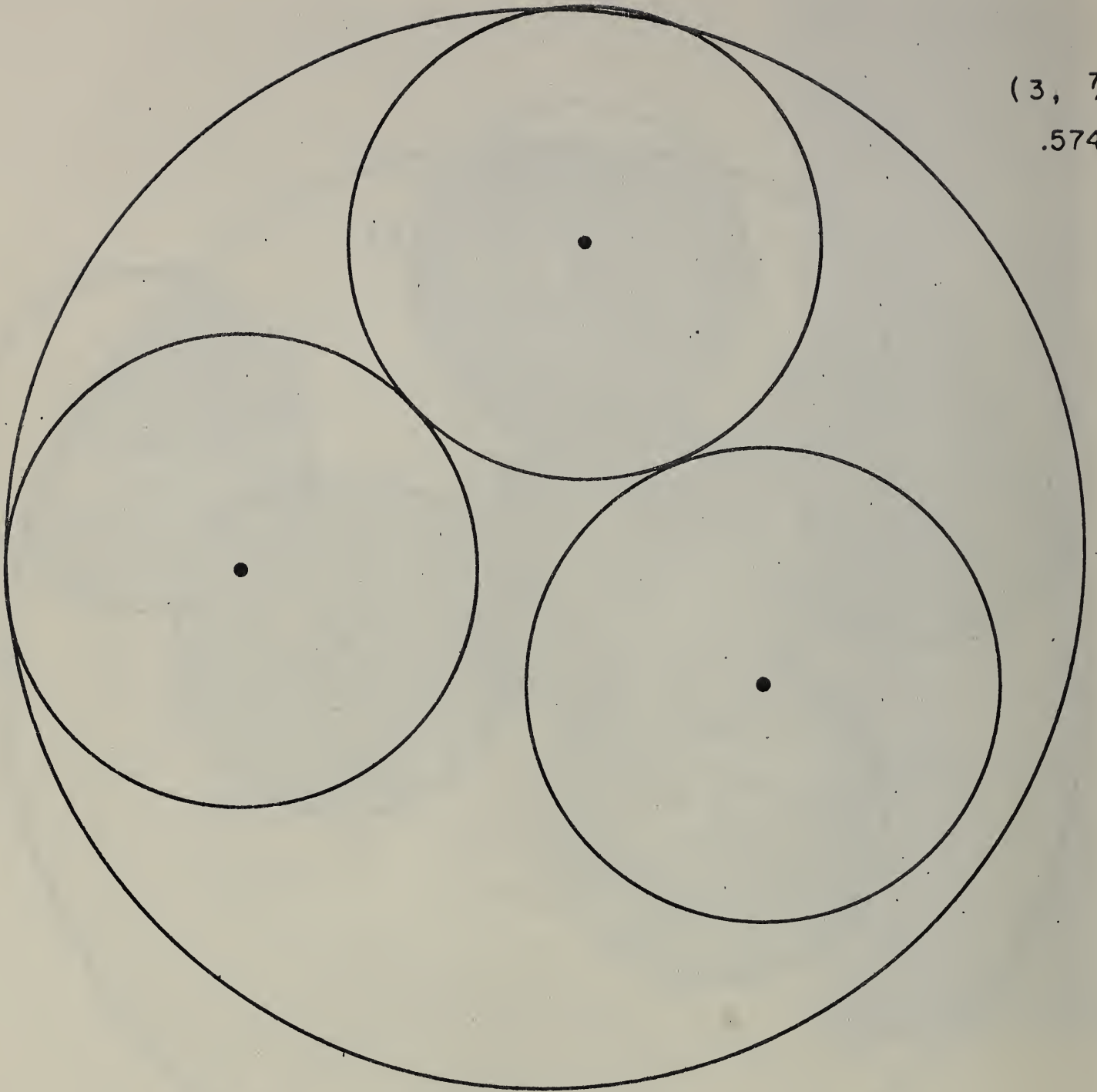


FIG. 2 LOOSE PACKING OF THREE DISCS

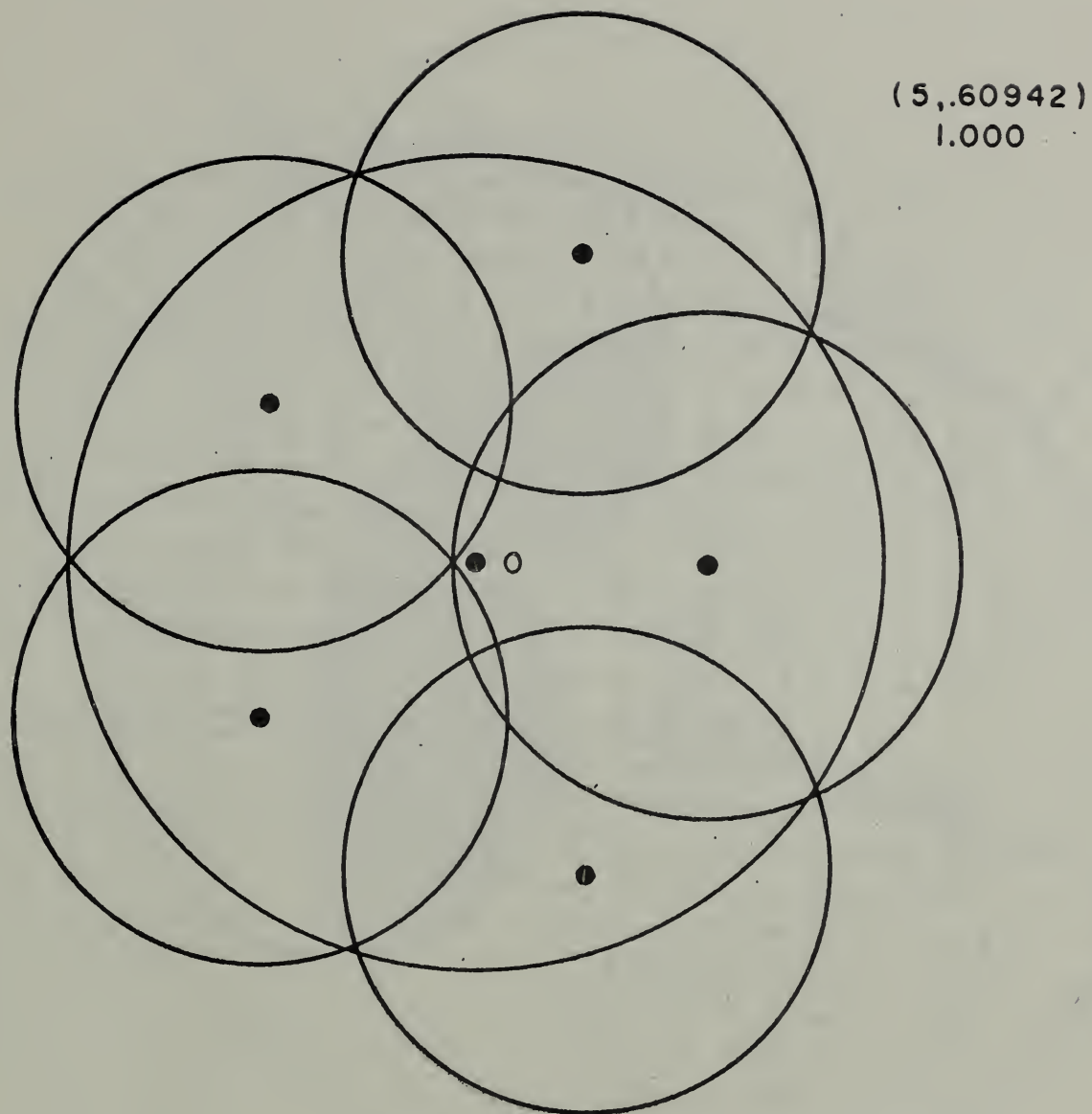


FIG. 3 NEVILLE'S FIVE DISC COVERING

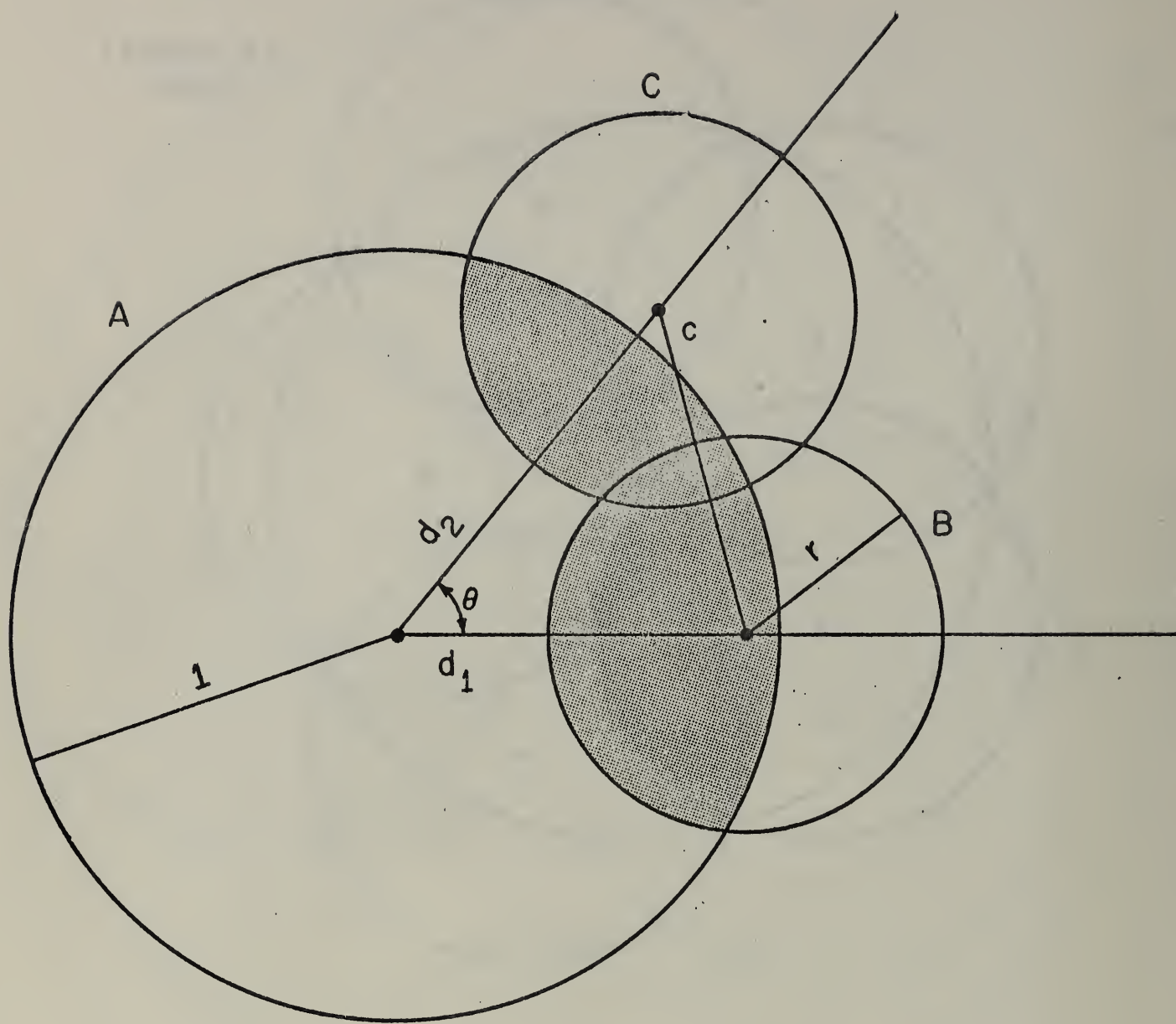


FIG. 4 PARAMETERS IN THE TWO DISC CASE

RATIO
314 / 793
.396

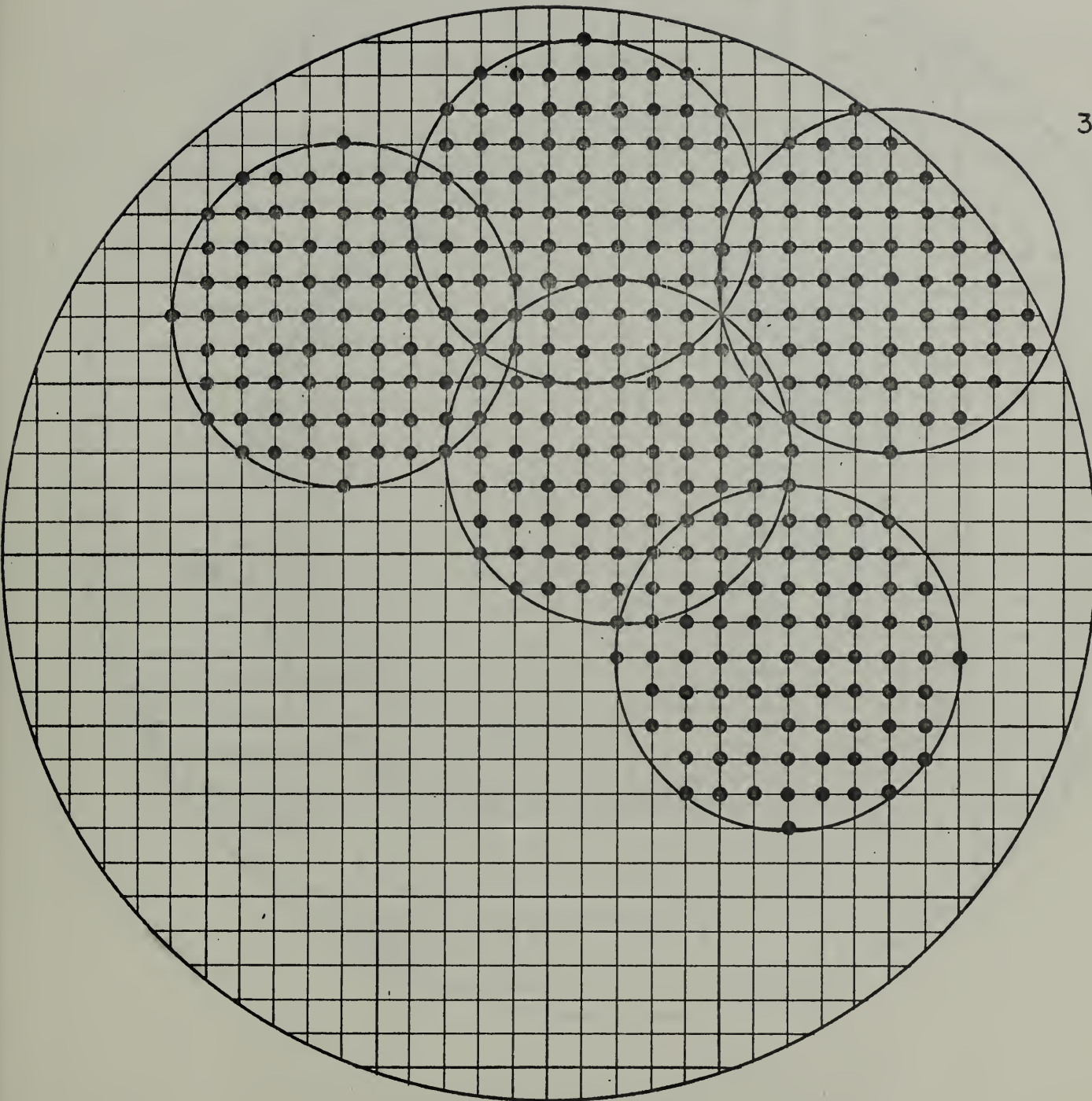


FIG.5 APPROXIMATING THE COVERAGE

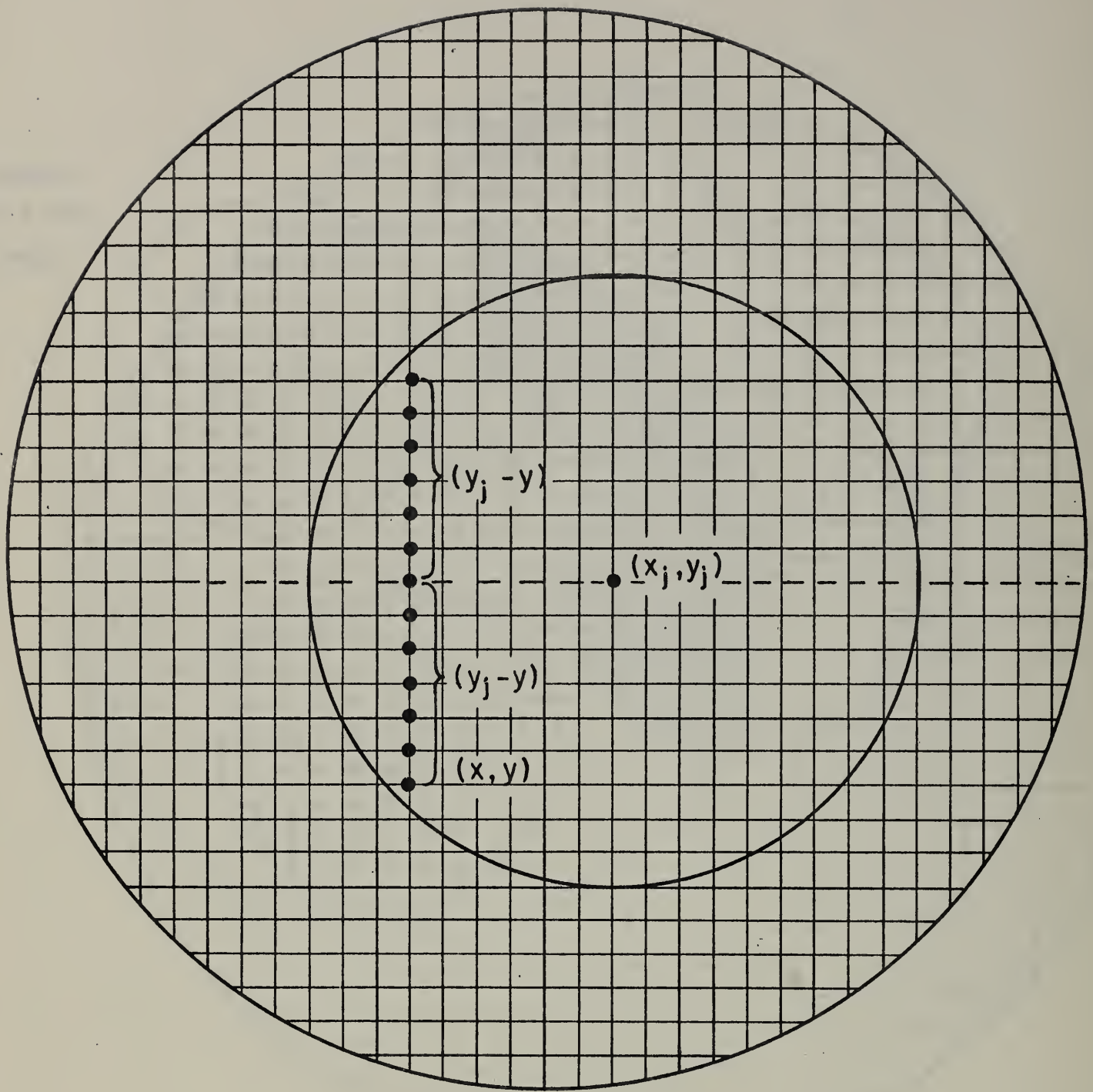


FIG. 6 REFLECTION SHORT - CUT

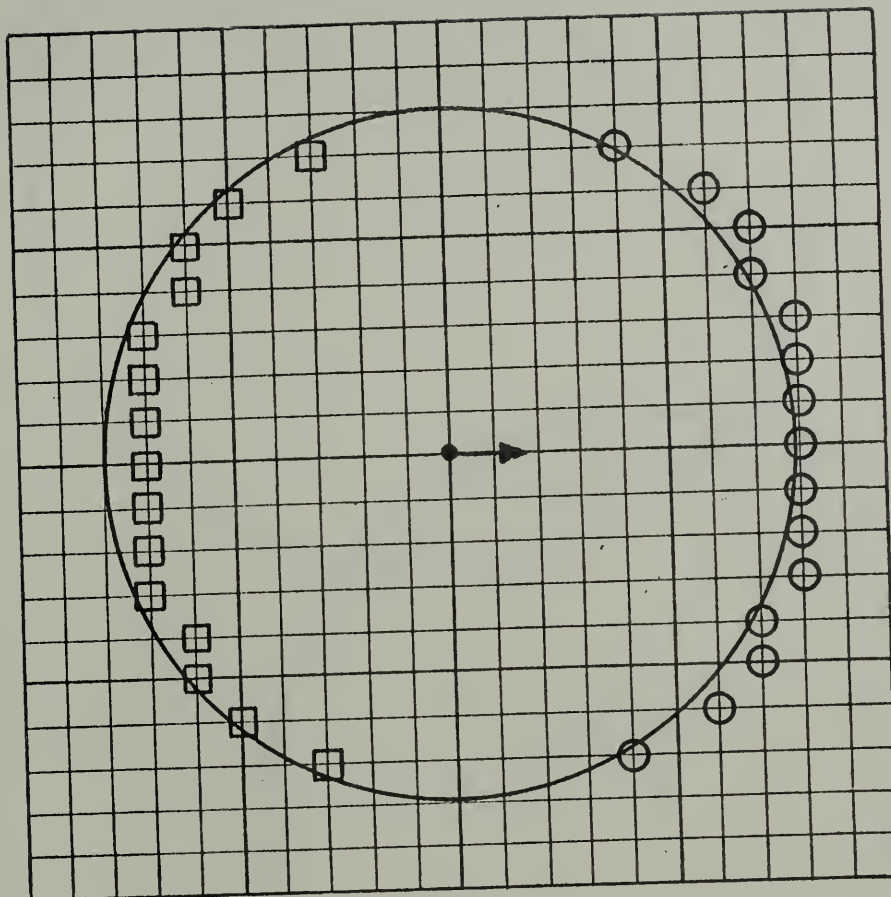
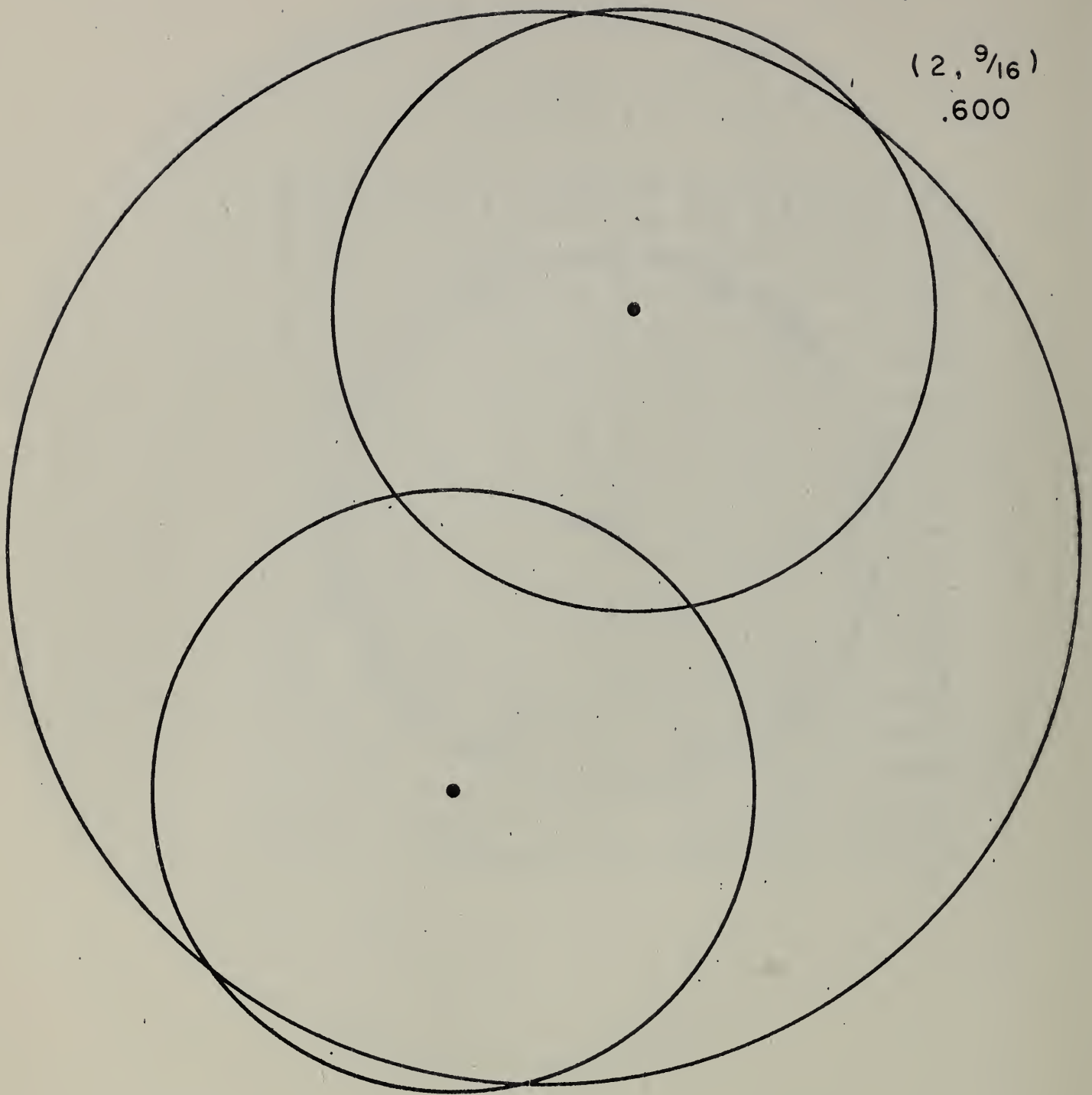
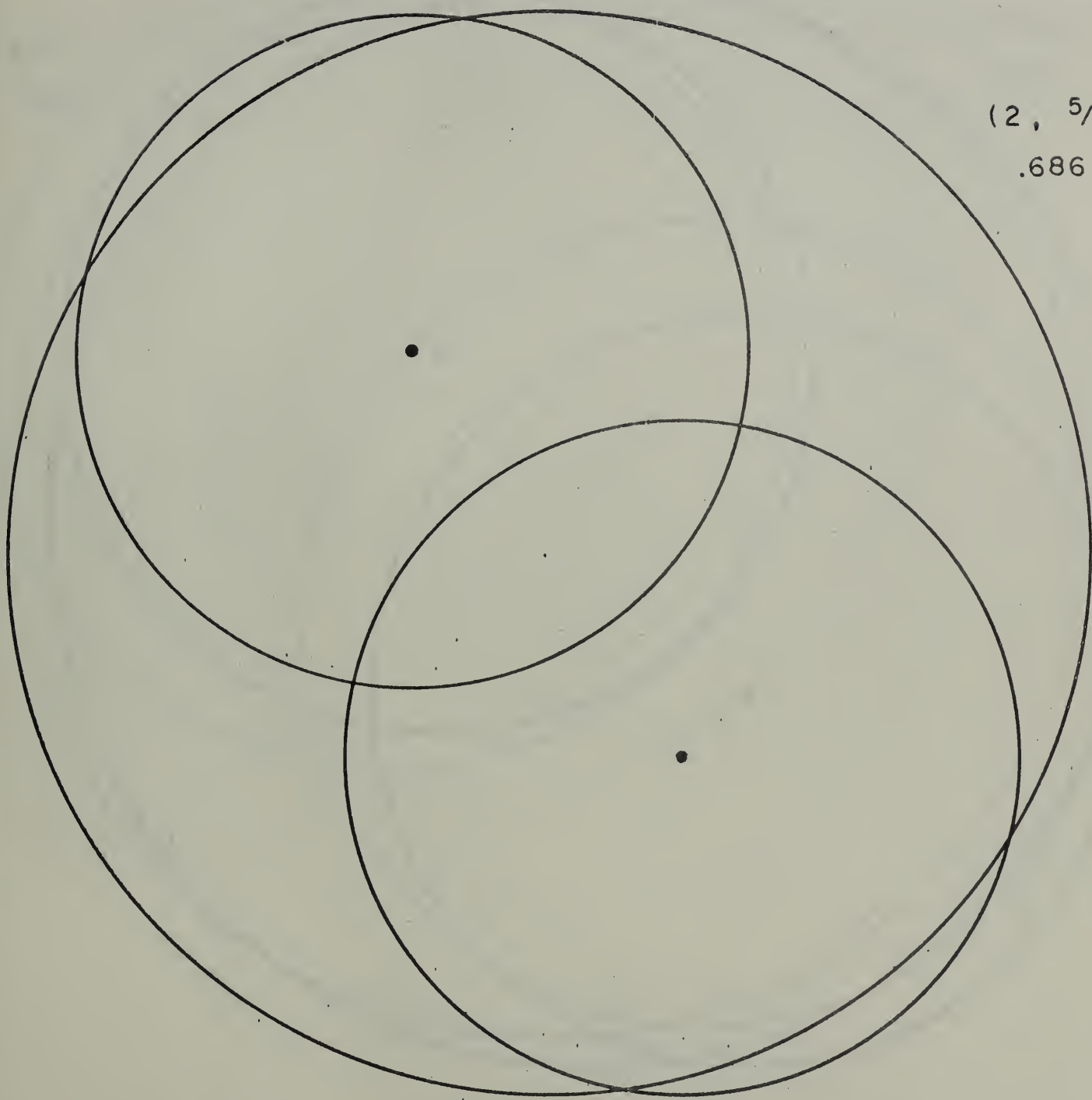


FIG. 7 POINTS TO BE SCANNED



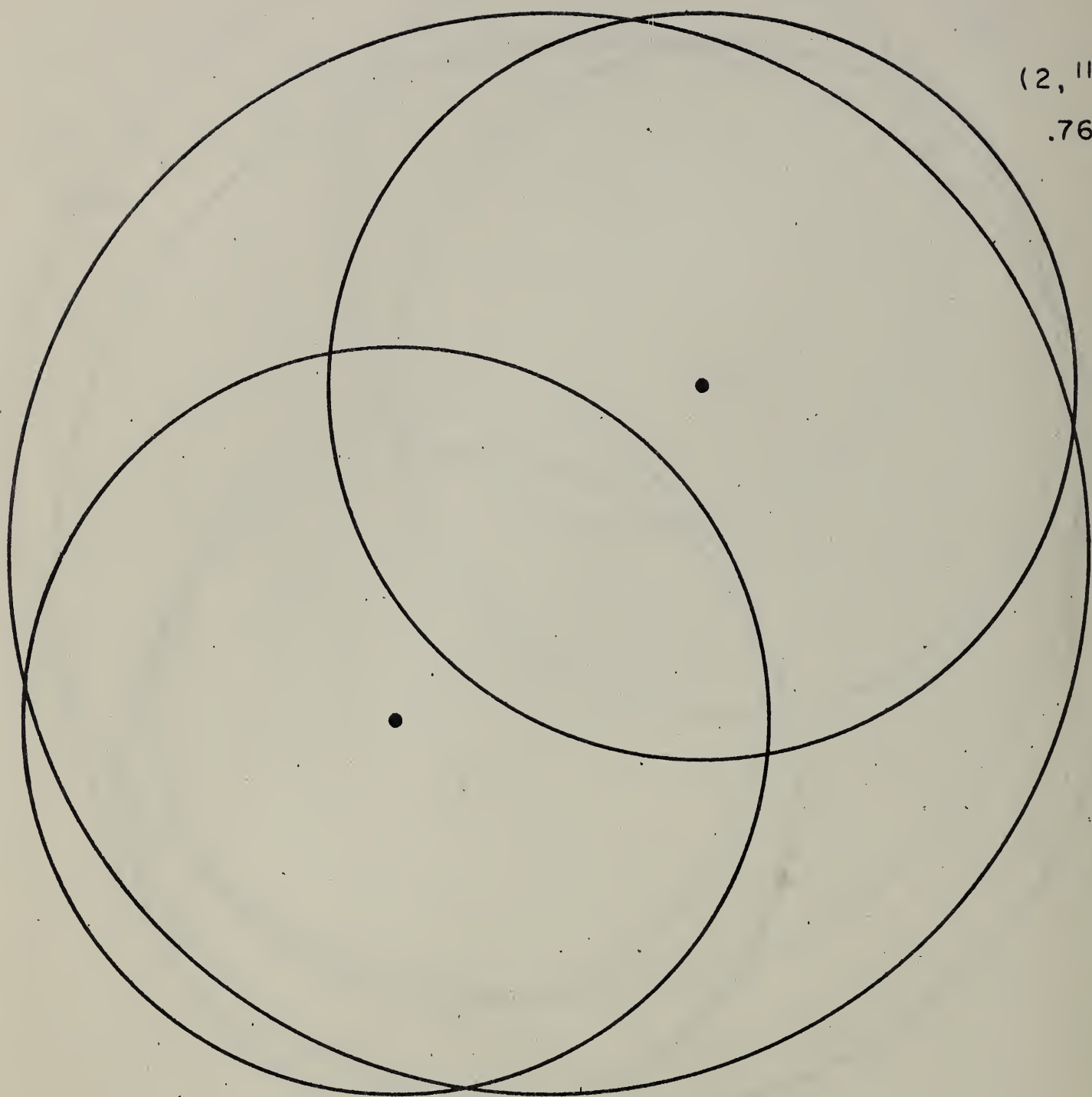
$(2, \frac{9}{16})$
.600

FIG. 8



$(2, \frac{5}{8})$
.686

FIG. 9



$(2, 11/16)$

.762

FIG. 10

$(2, \frac{3}{4})$

.829

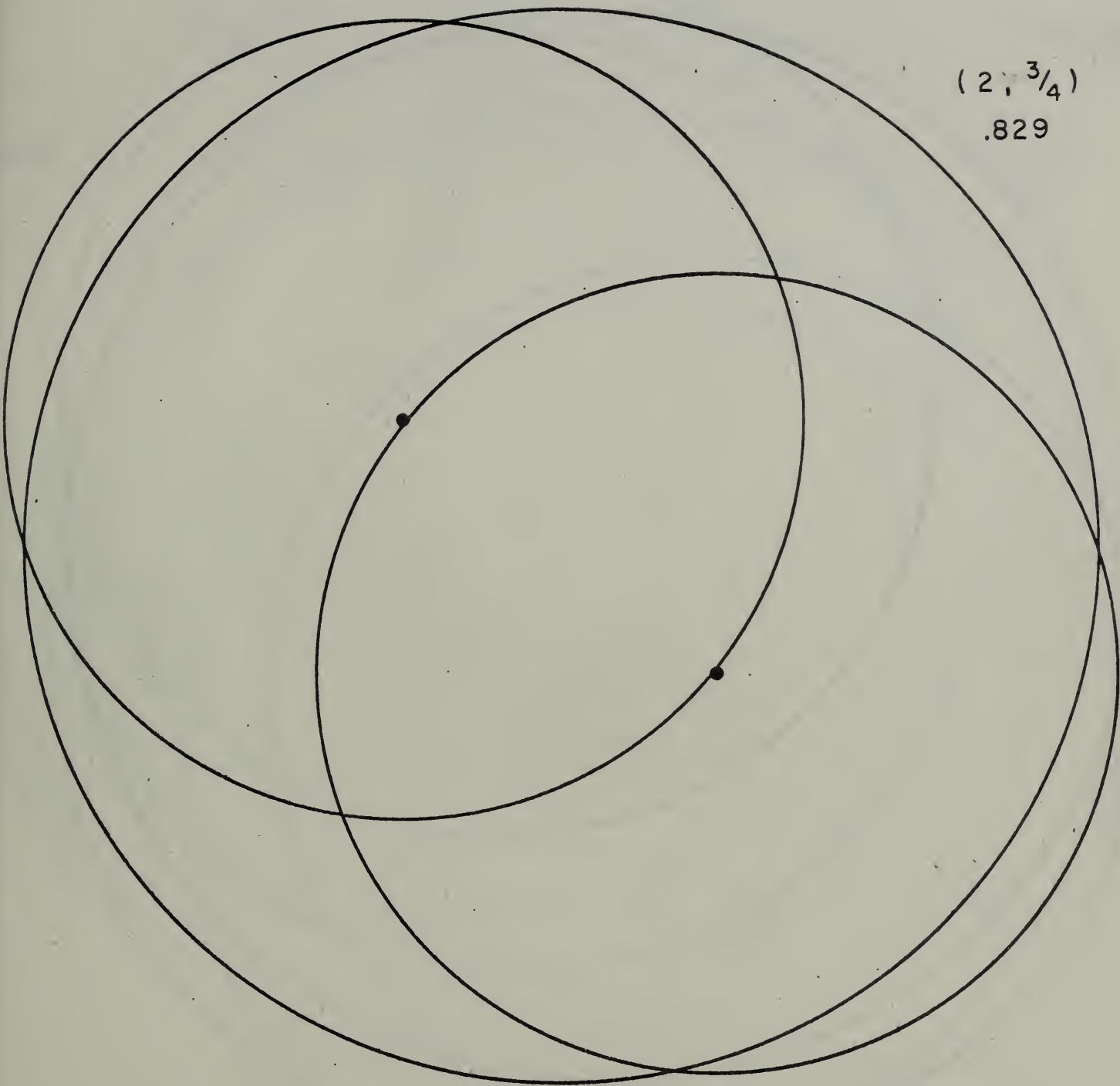
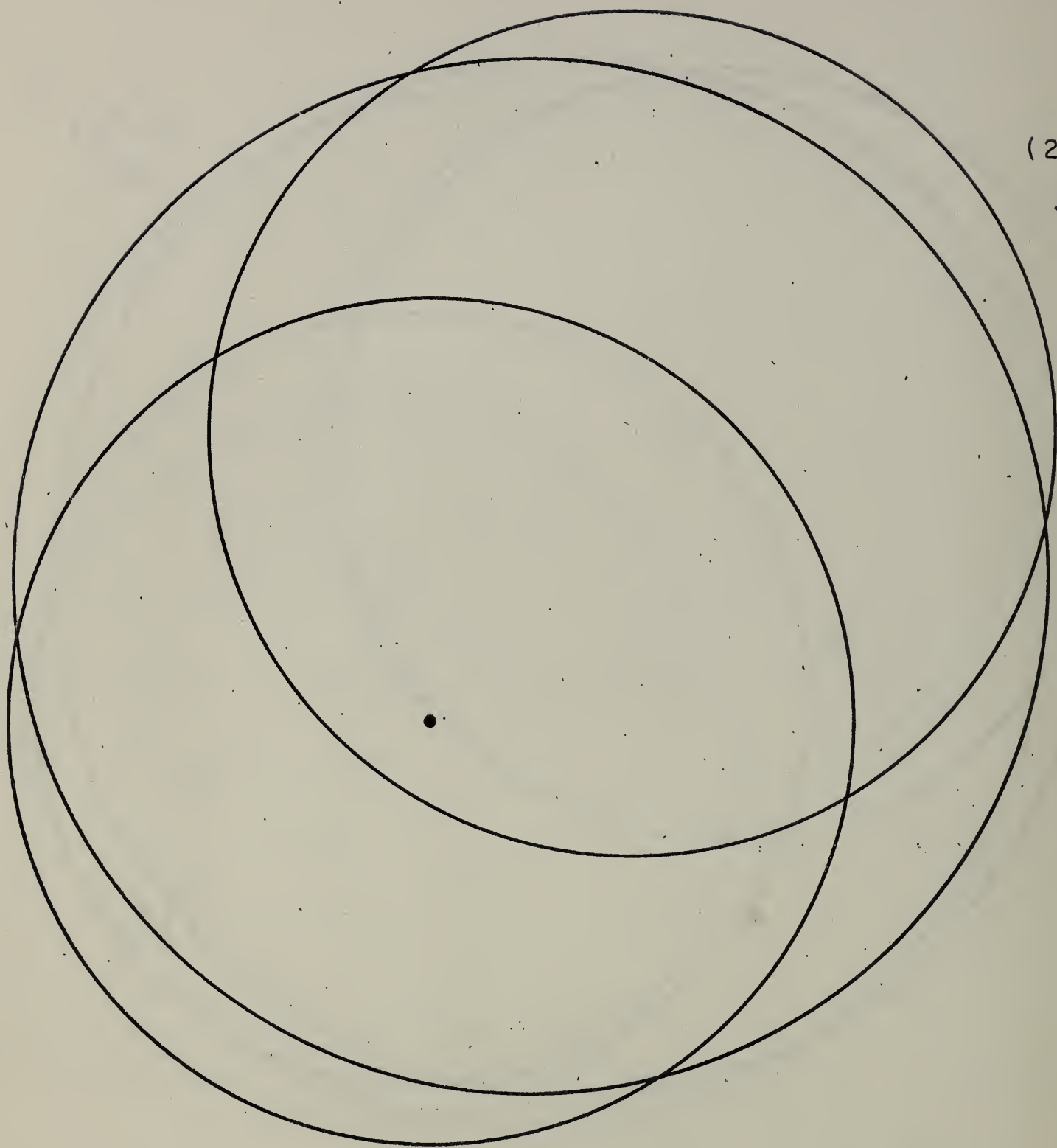
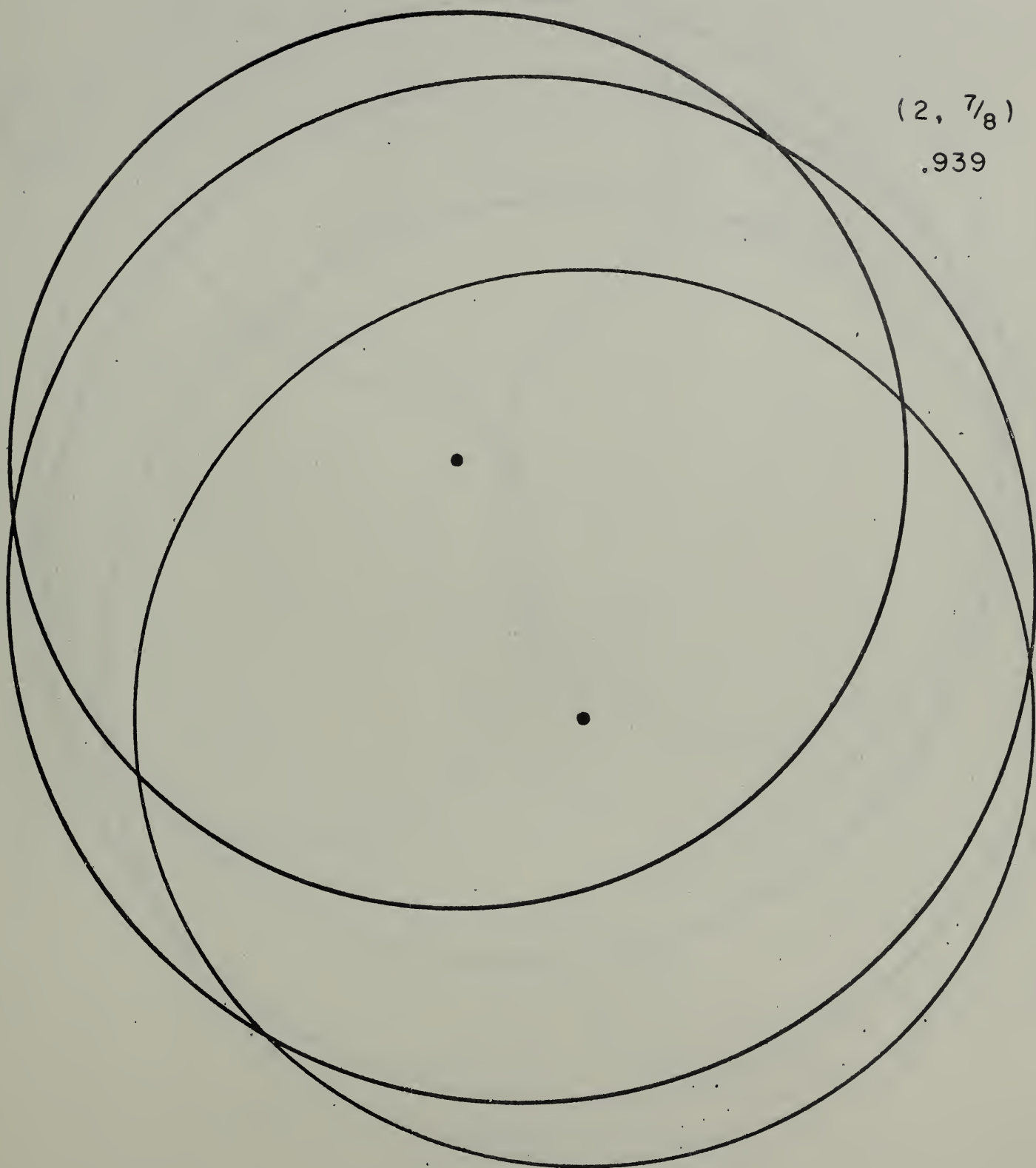


FIG. II



(2, $\frac{13}{16}$)
.889

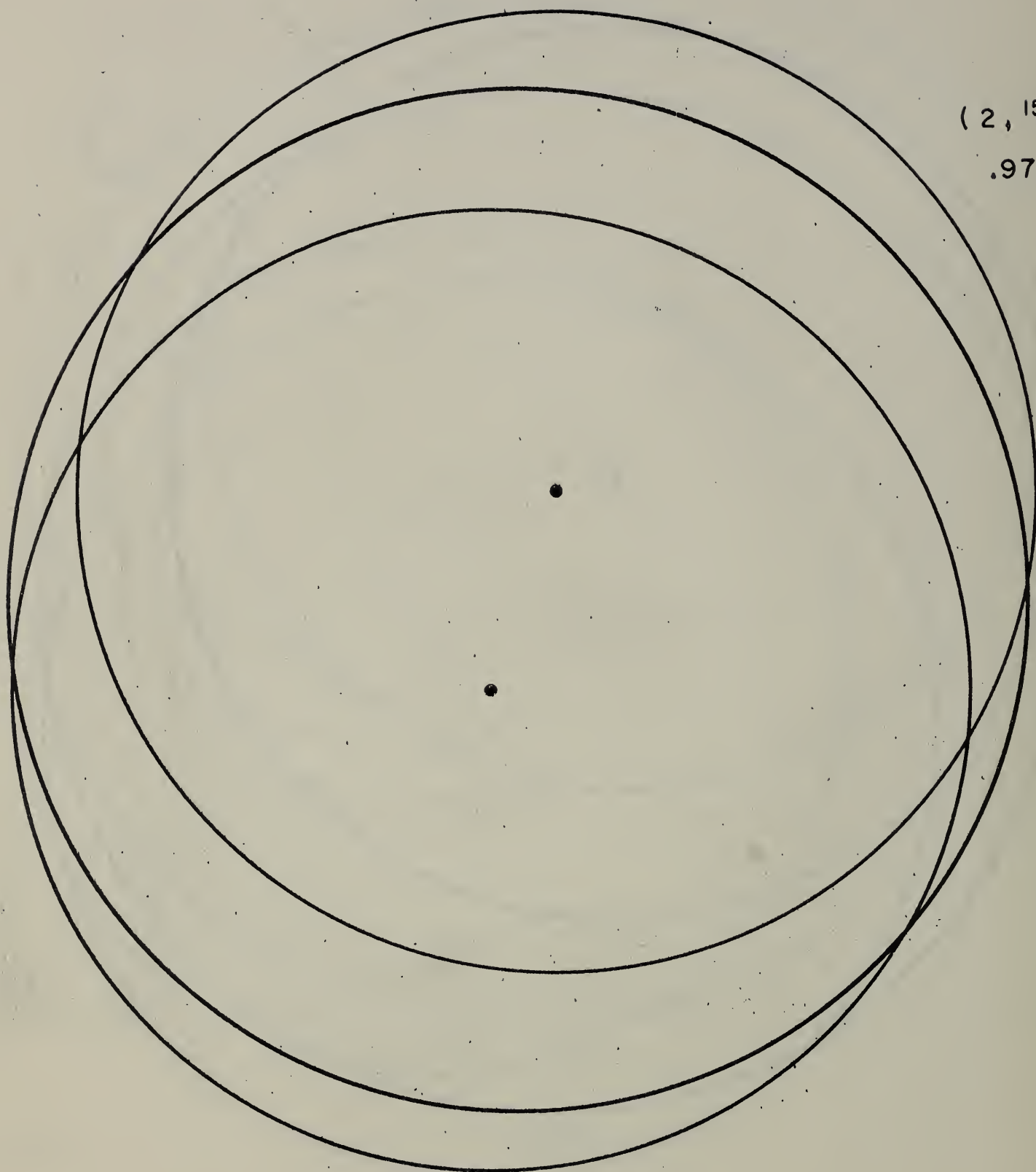
FIG. 12



$(2, \frac{7}{8})$

.939

FIG. 13



$(2, \frac{15}{16})$
.979

FIG. 14

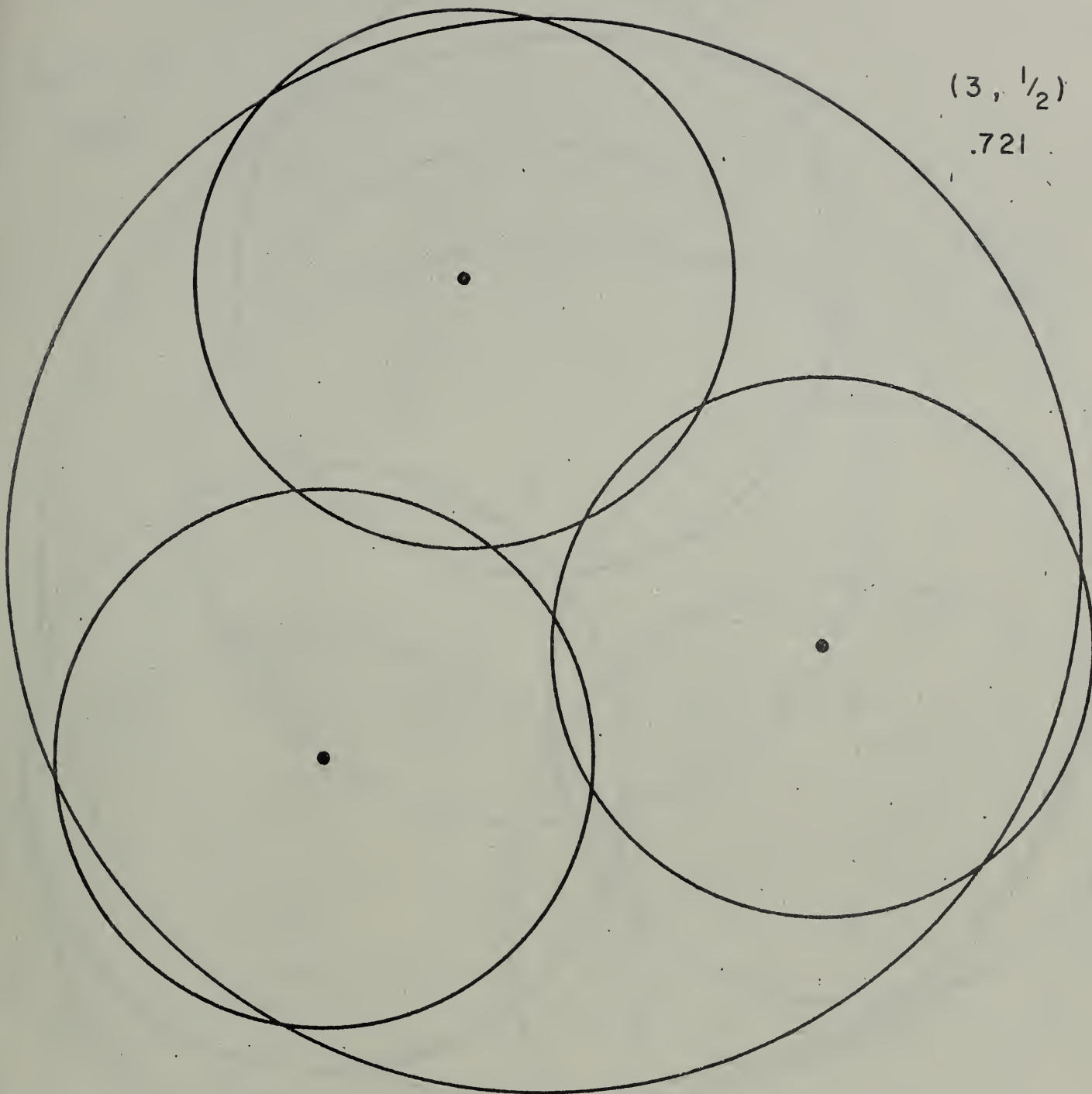


FIG. 15

$(3, \frac{9}{16})$

.815

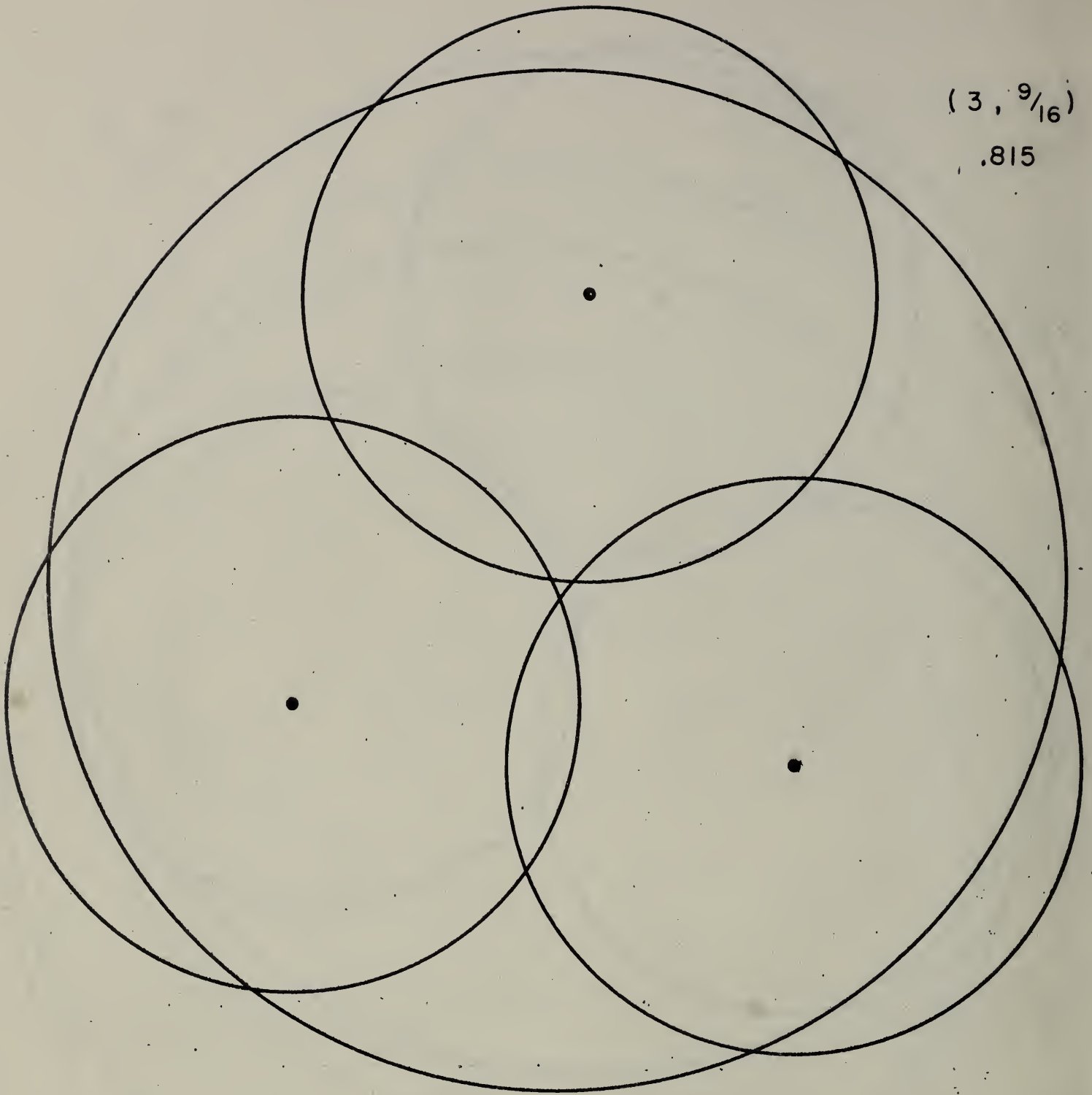


FIG. 16

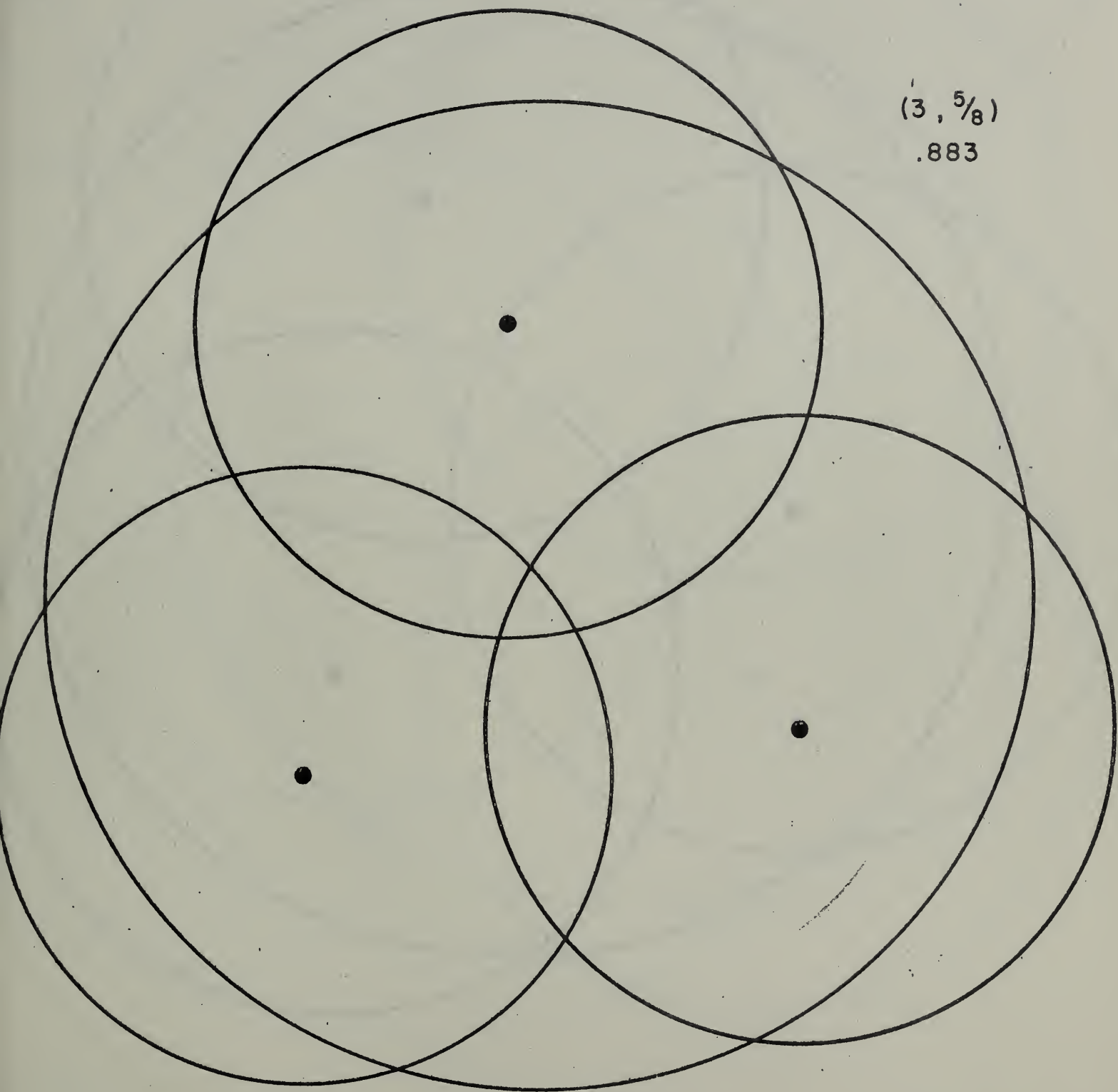


FIG. 17

$(3, 11/16)$

.935

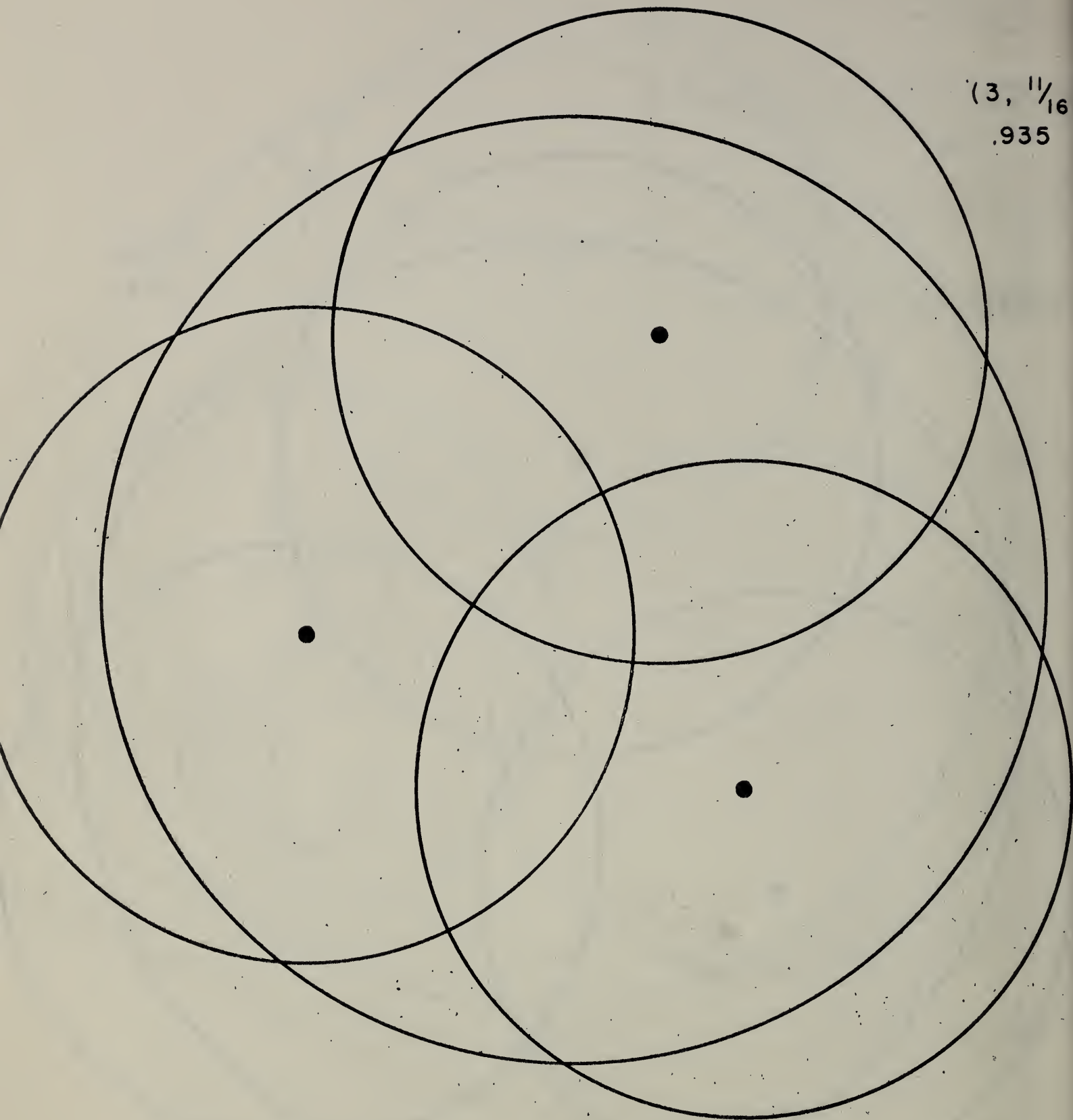


FIG. 18

$(3, \frac{3}{4})$
.972

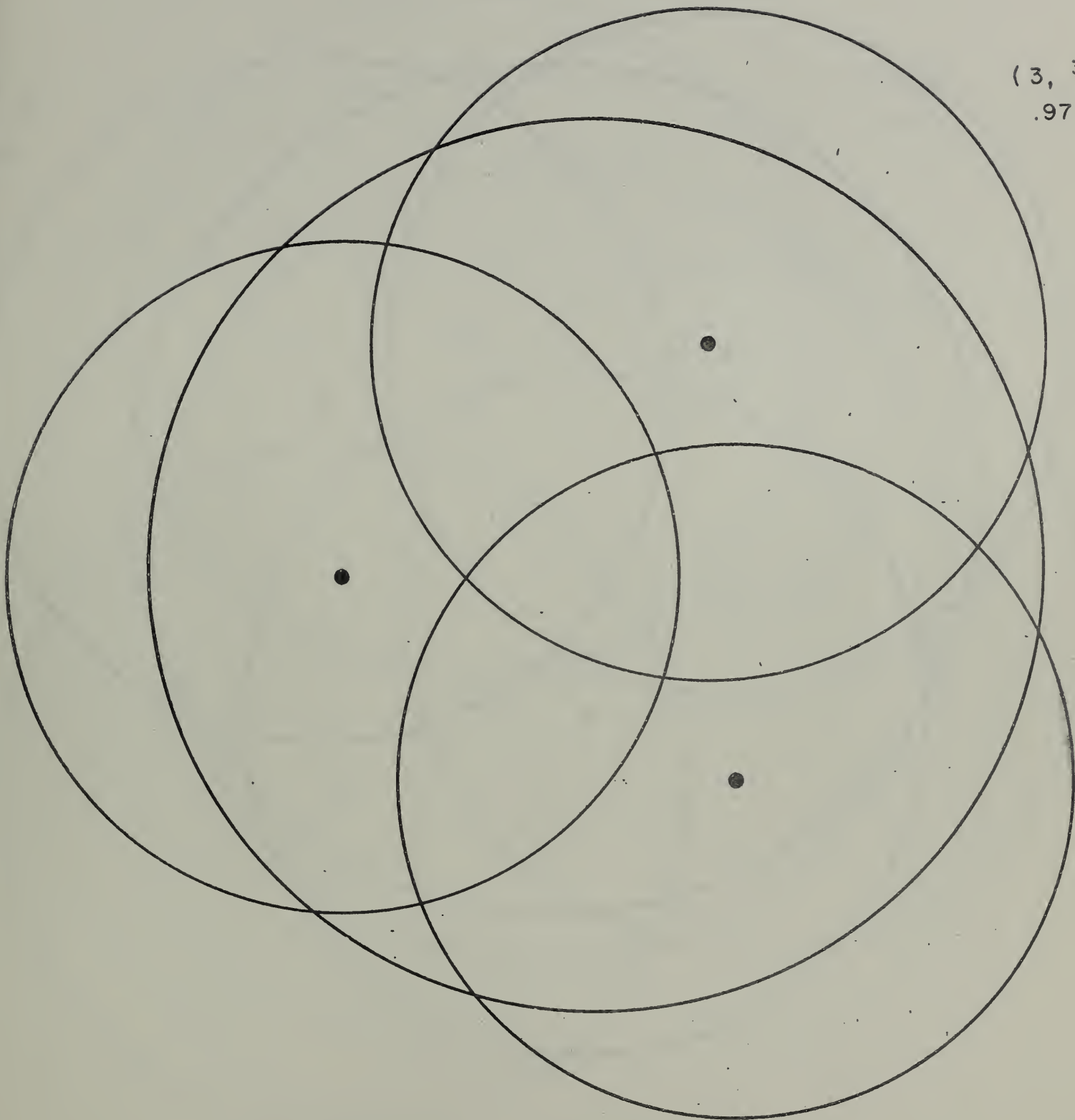


FIG. 19

(3, ¹³/₁₆)

:994

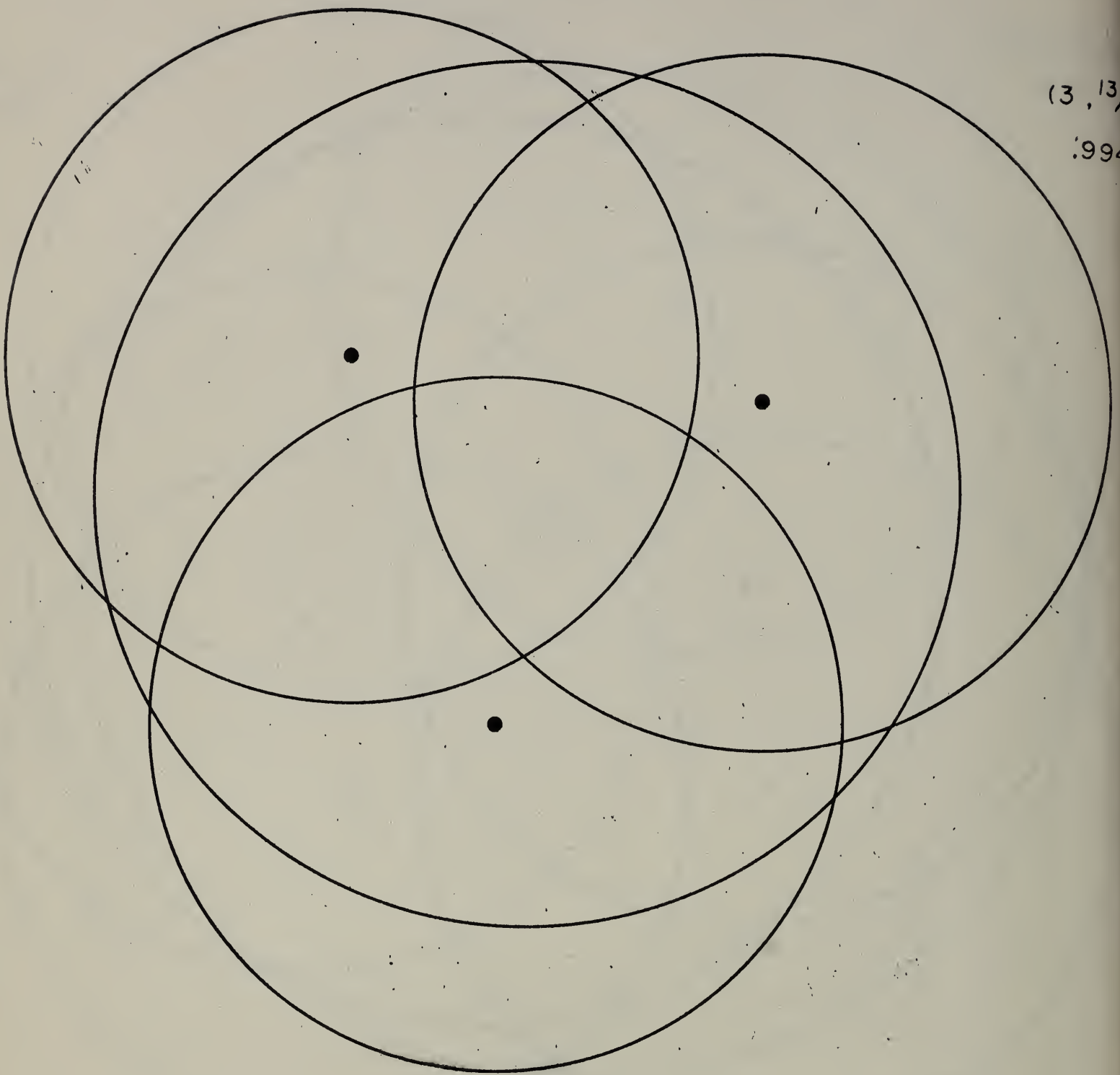


FIG. 20

$(3, \frac{7}{8})$

1.000

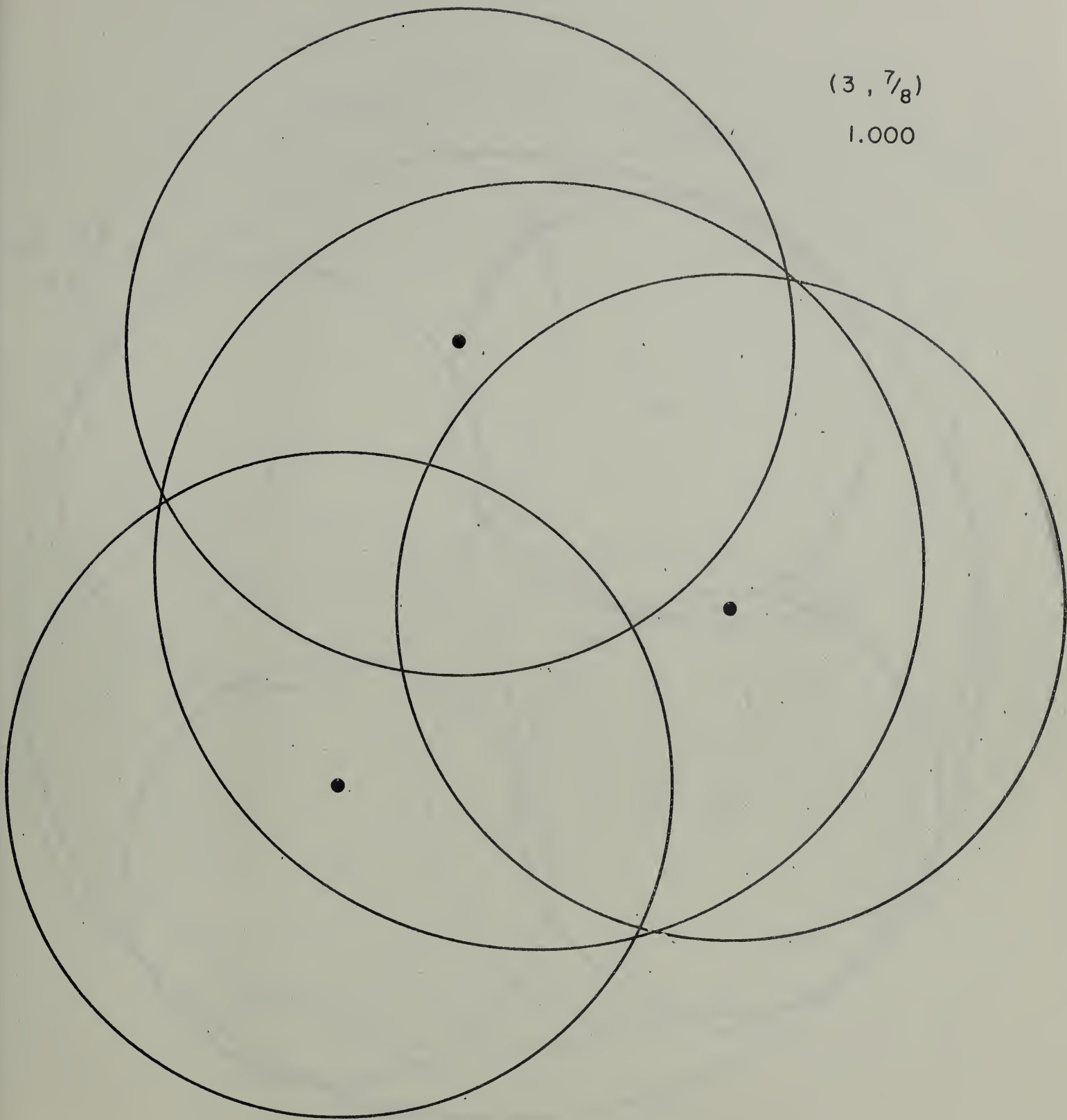


FIG. 21

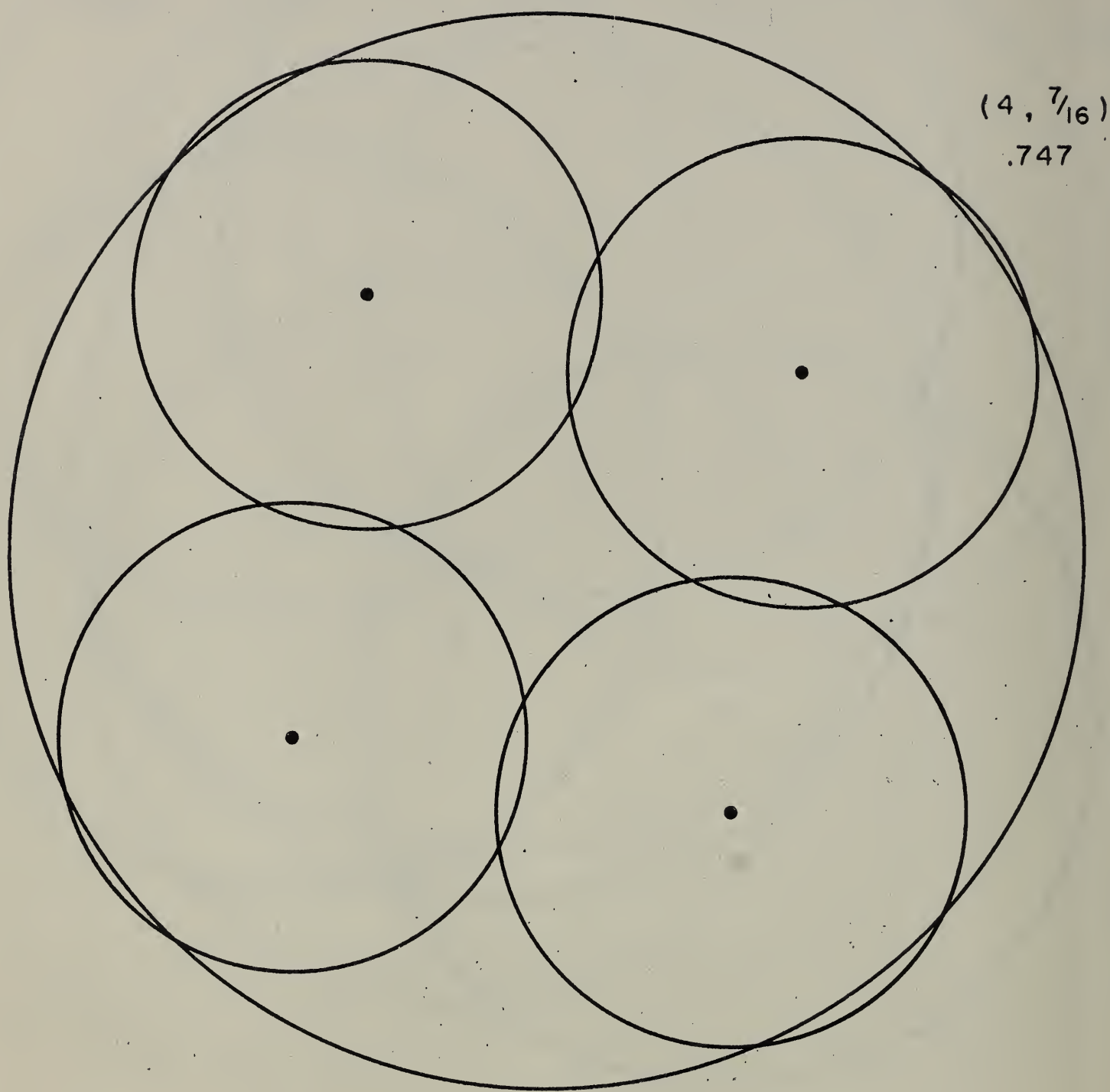


FIG.22

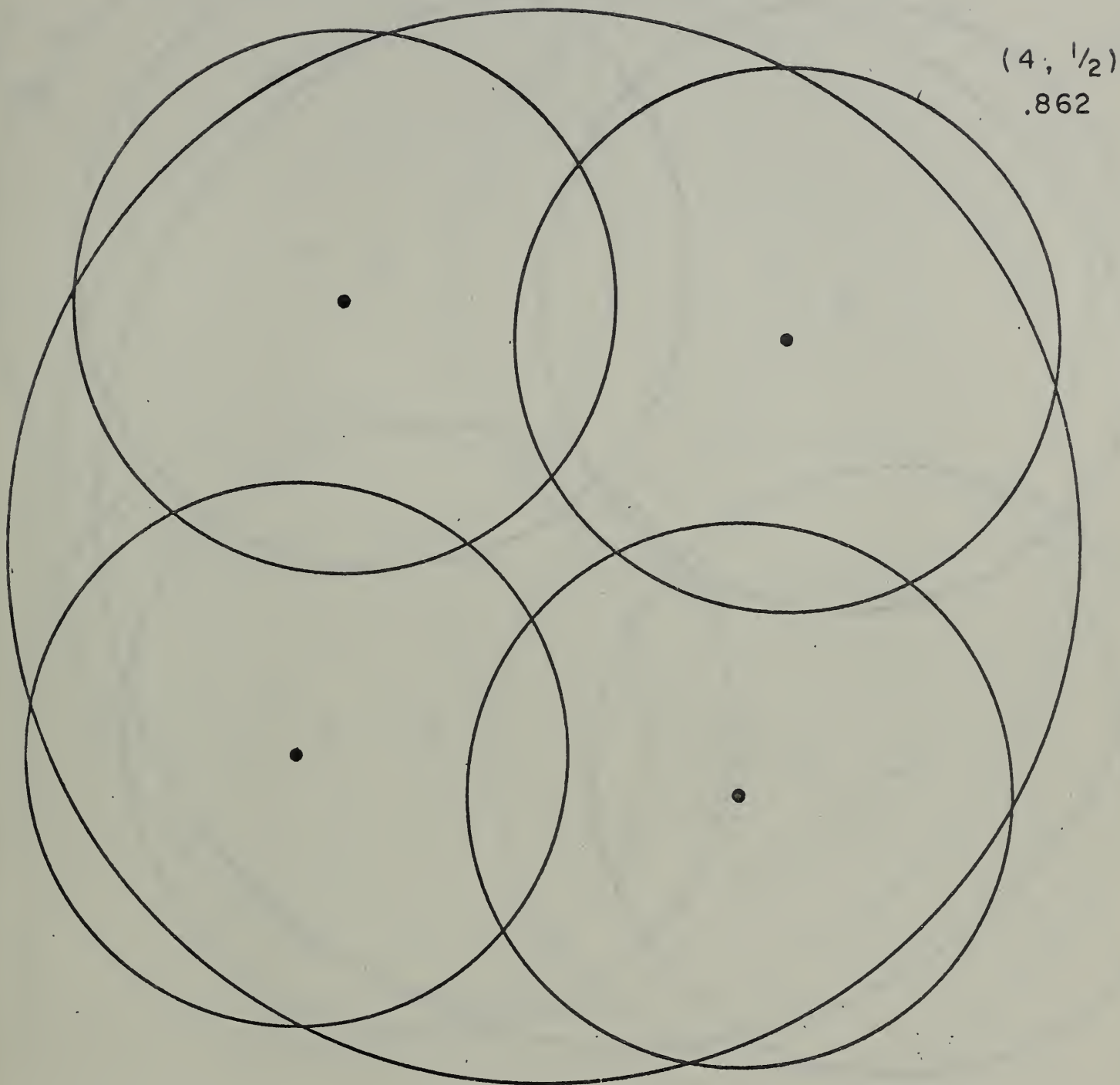
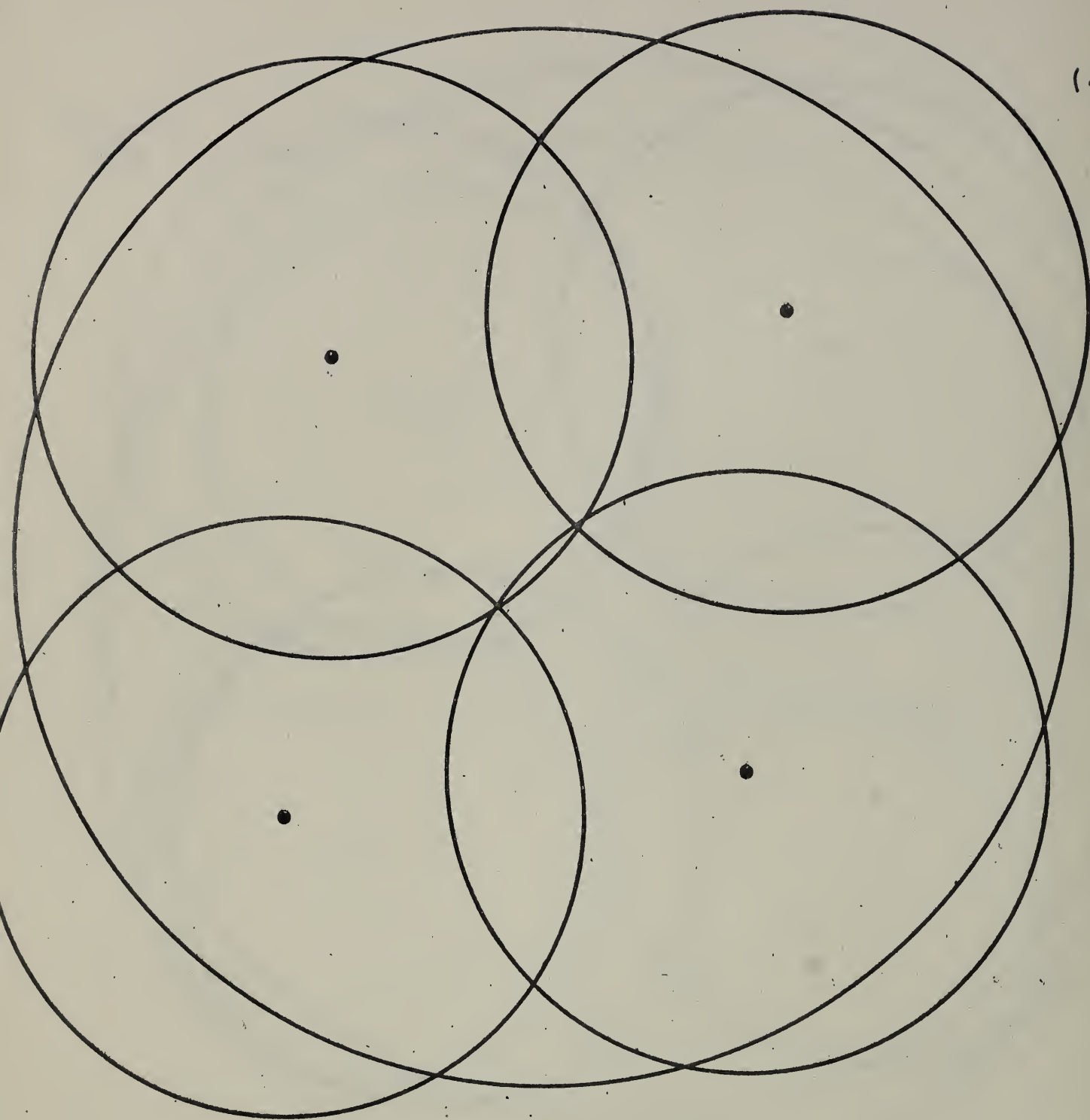


FIG. 23



$(4, \frac{9}{16})$

.937

FIG. 24

$(4, \frac{5}{8})$
.982

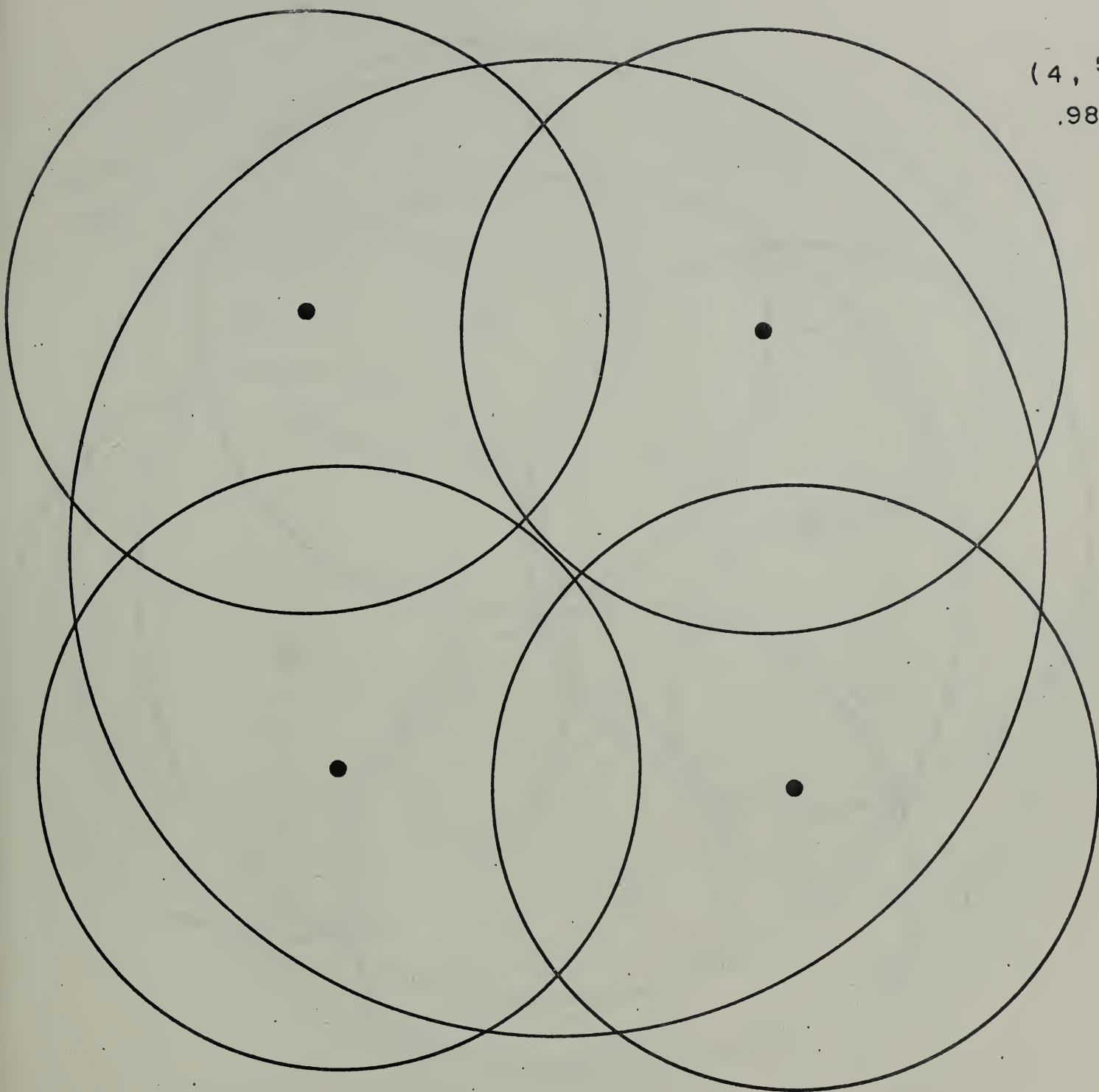


FIG. 25

$(4, 11/16)$

.999

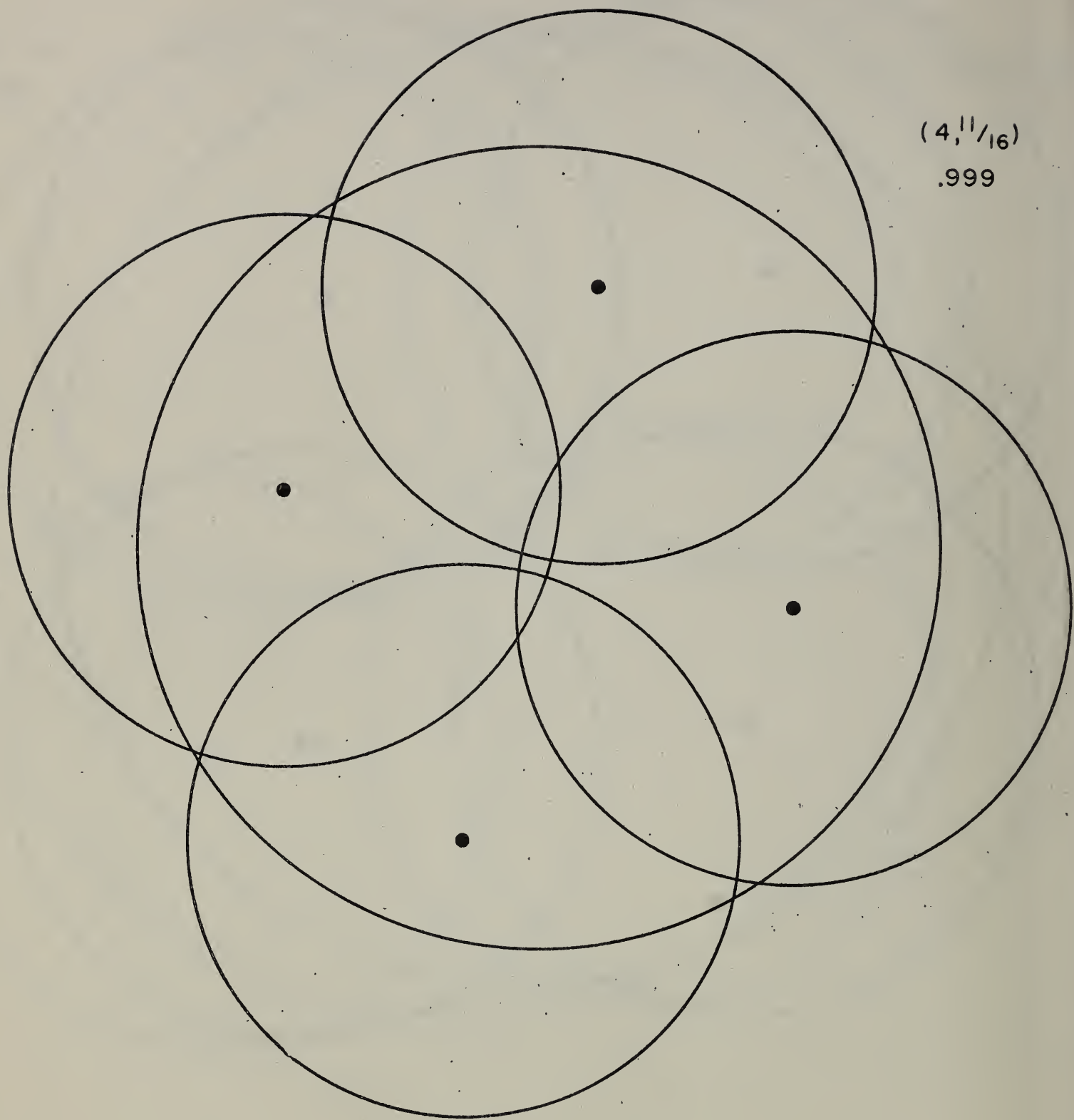


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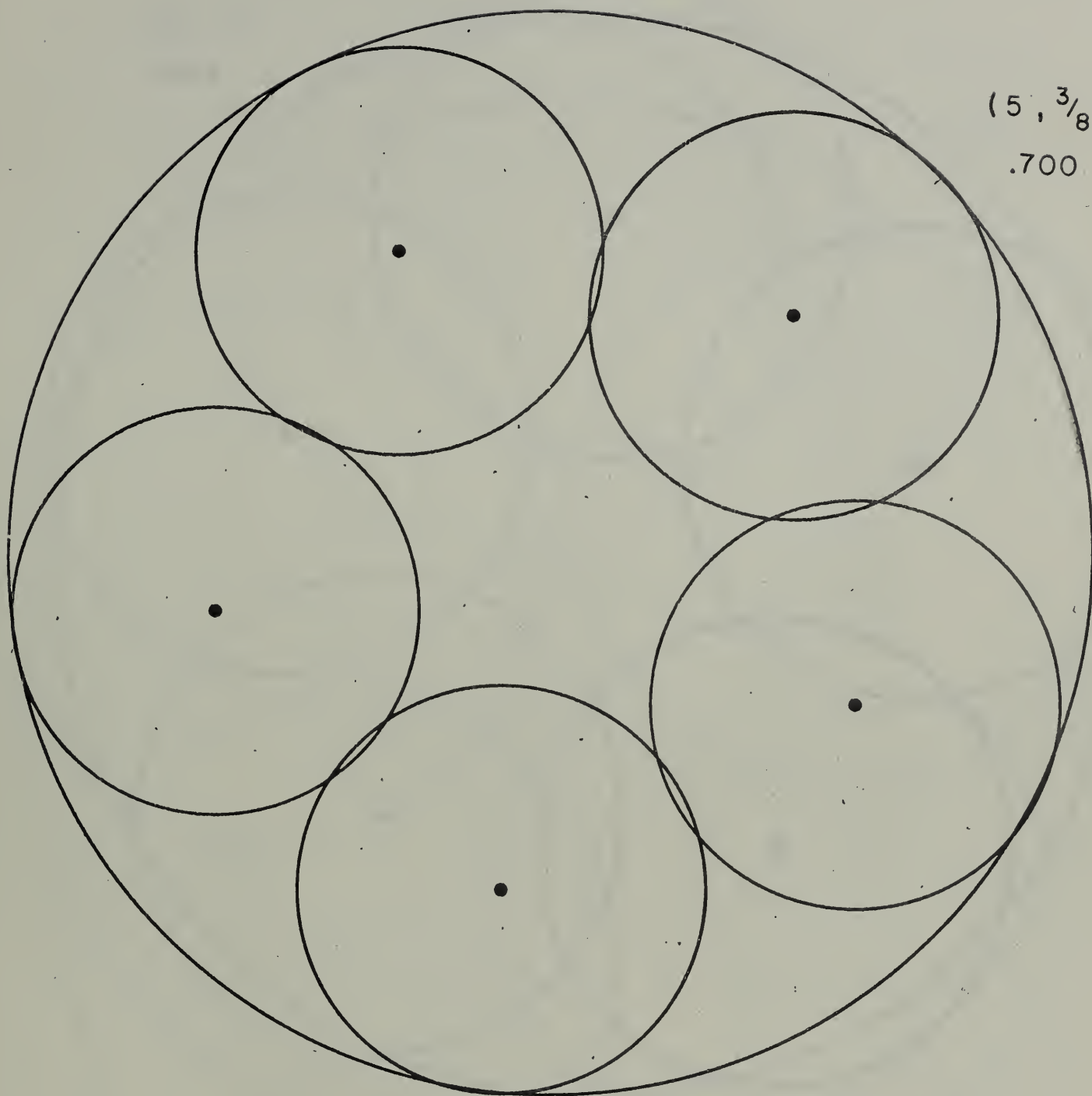


FIG. 27

$(5, \frac{7}{16})$
.846,

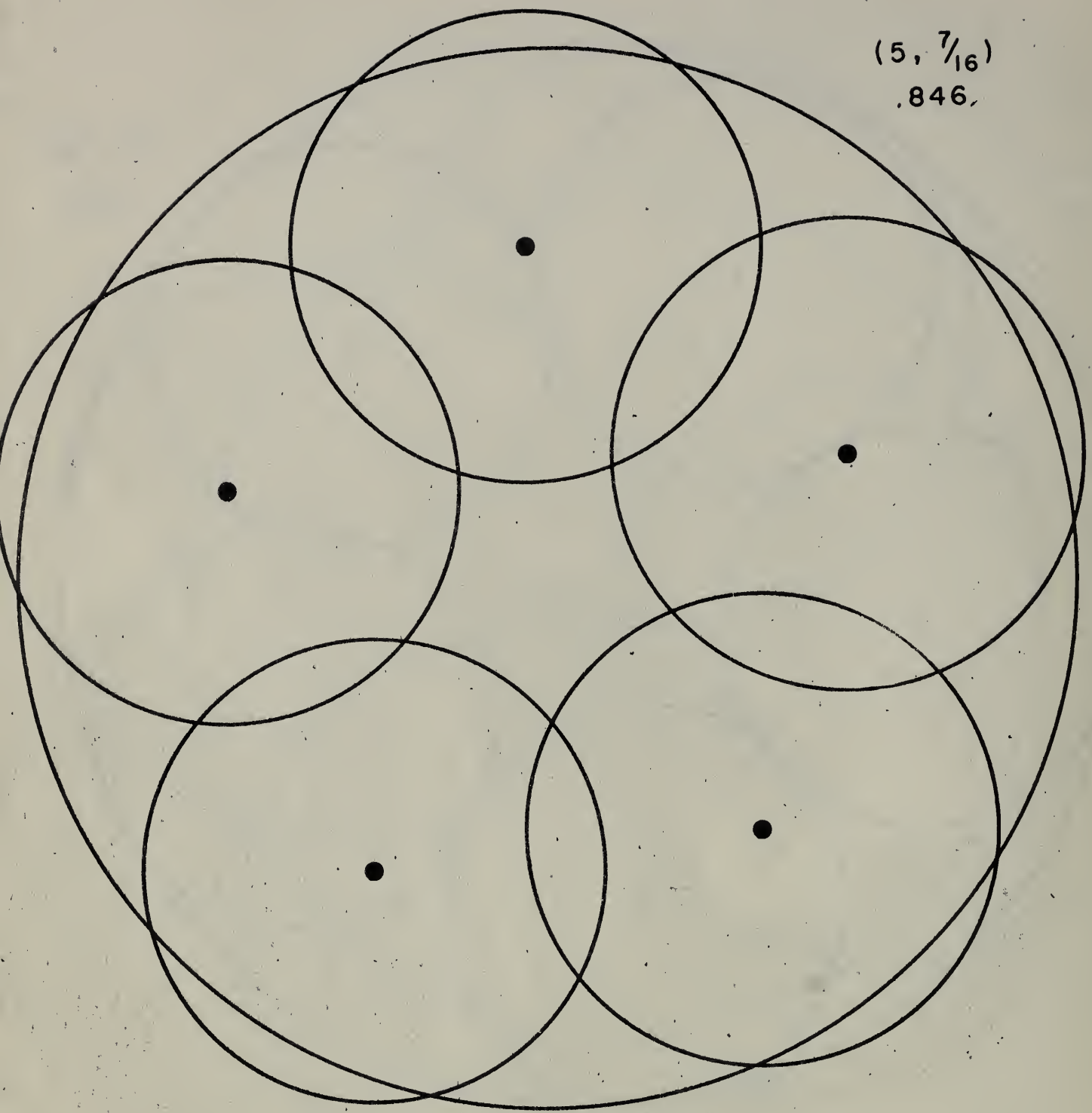


FIG.28

$(5, \frac{1}{2})$

.936

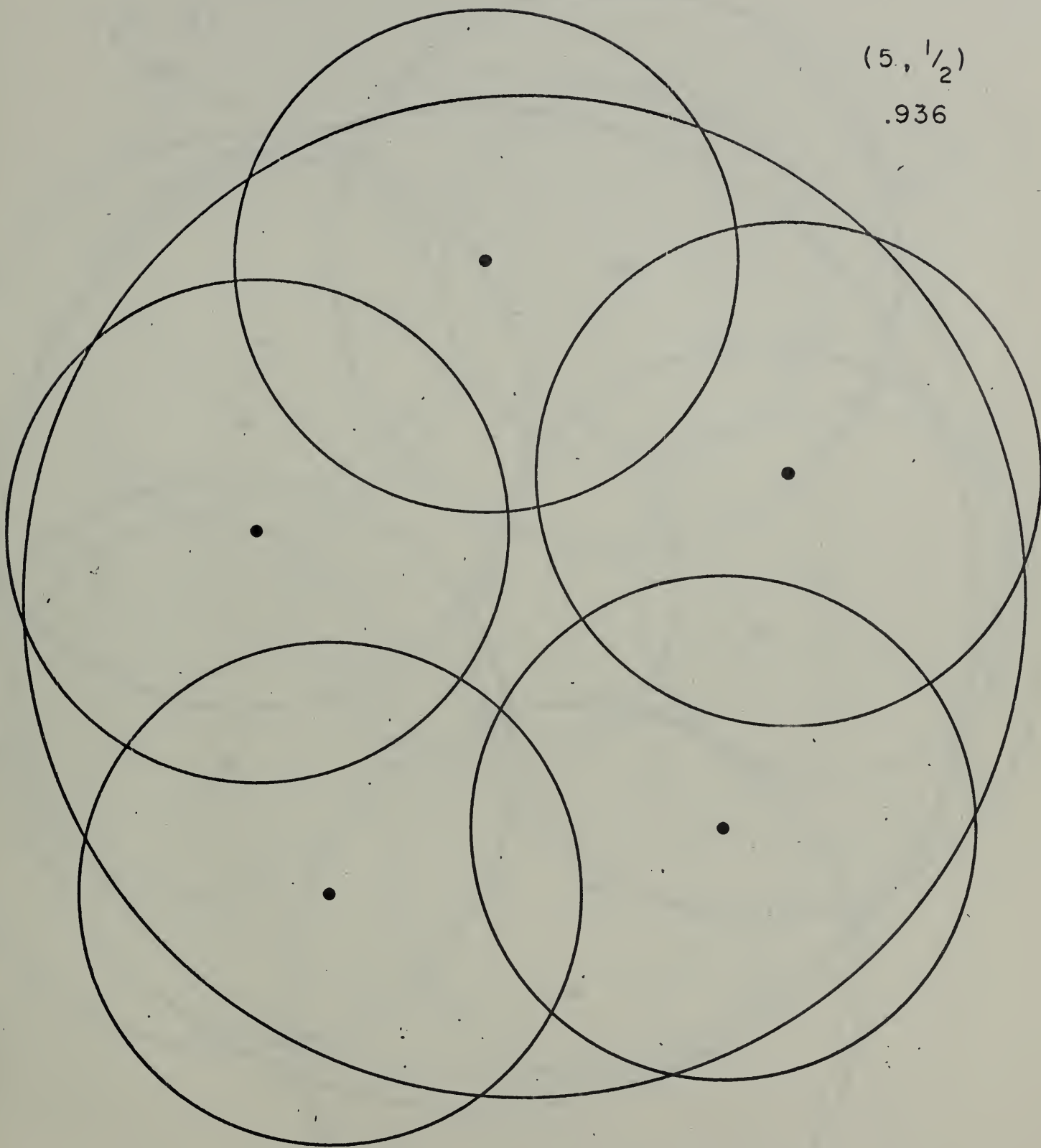


FIG.29

$(5, \frac{9}{16})$
.989

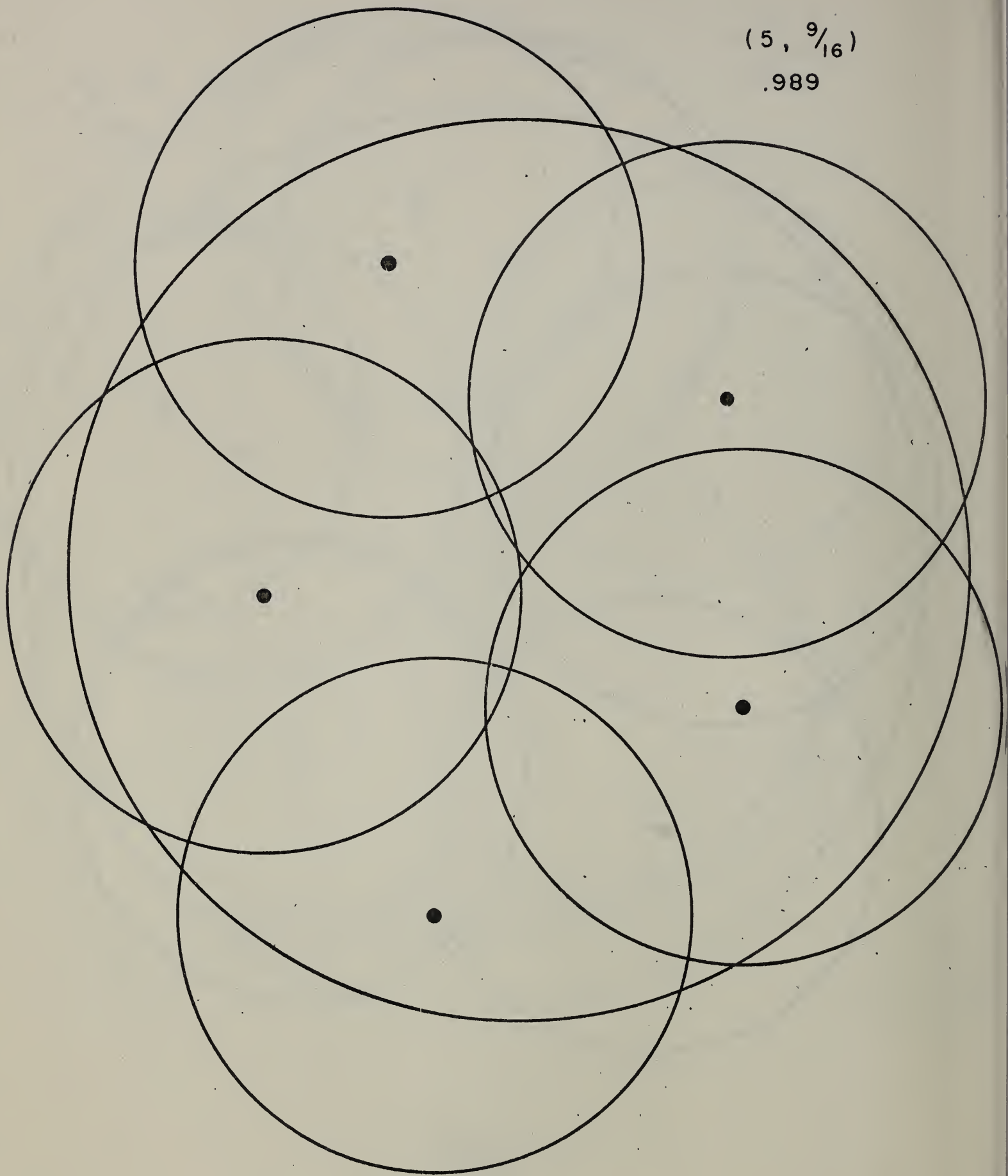


FIG. 30

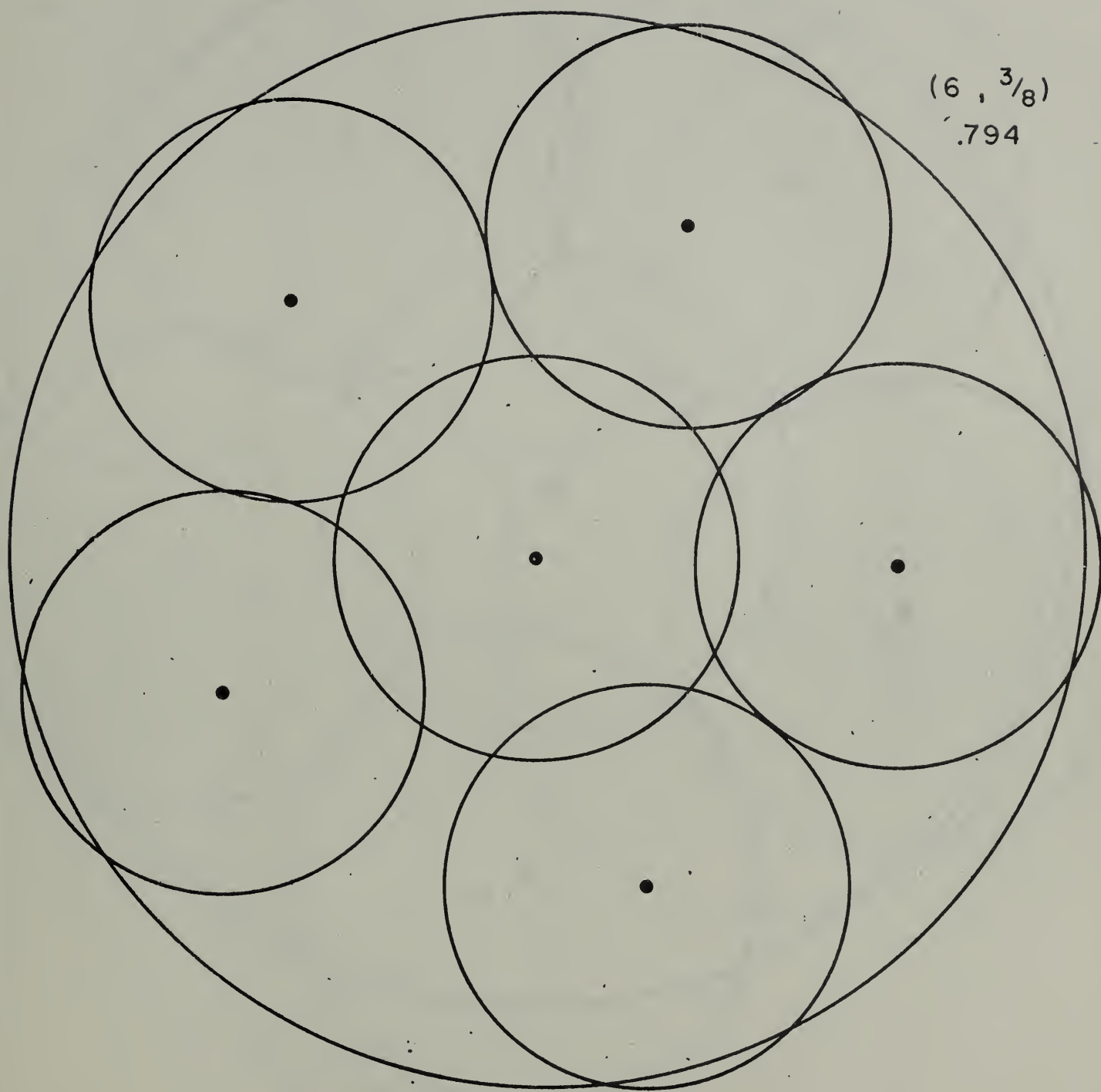


FIG. 31

$(6, \frac{7}{16})$
.915

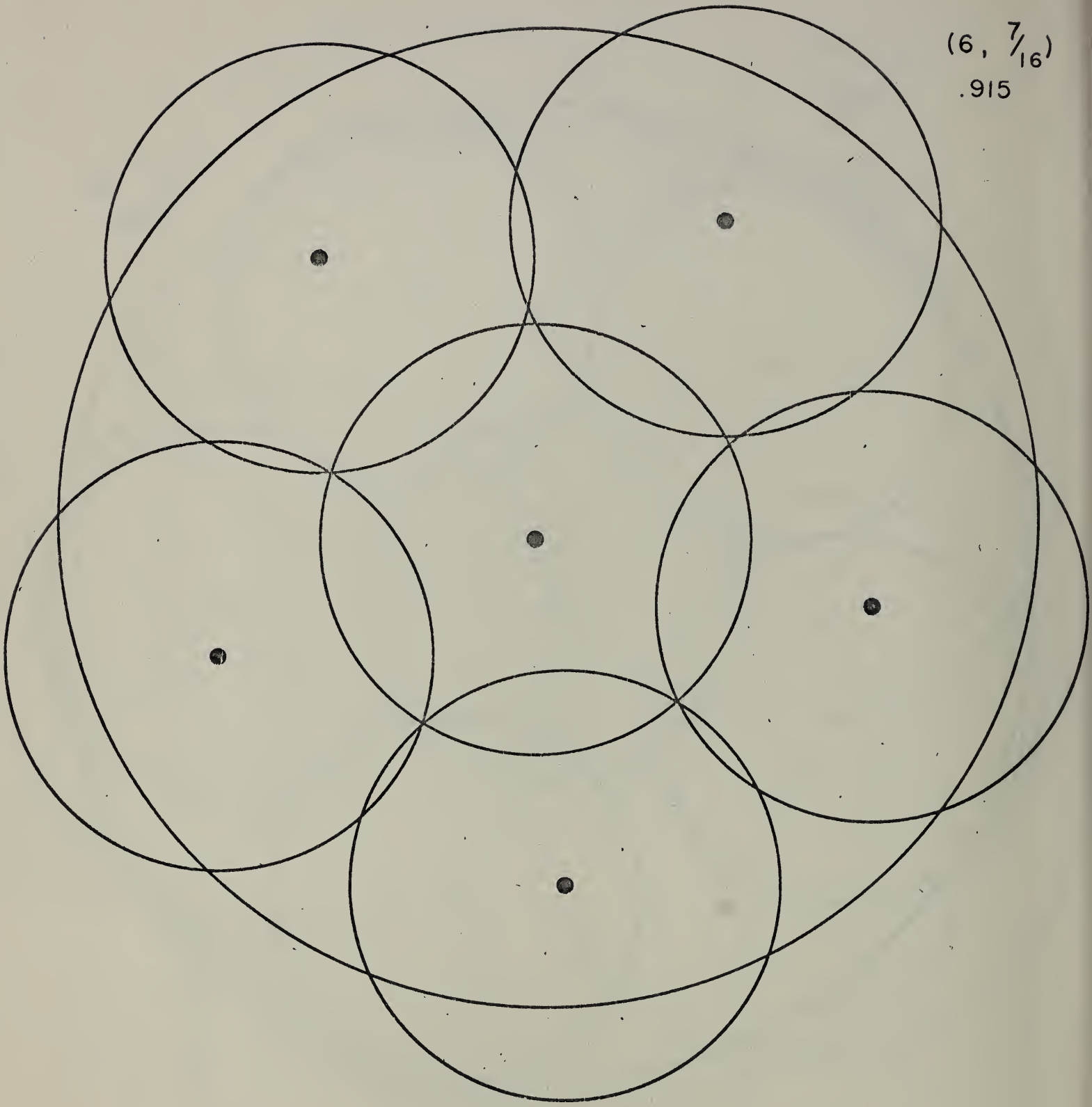


FIG. 32

$(6, \frac{1}{2})$
.979

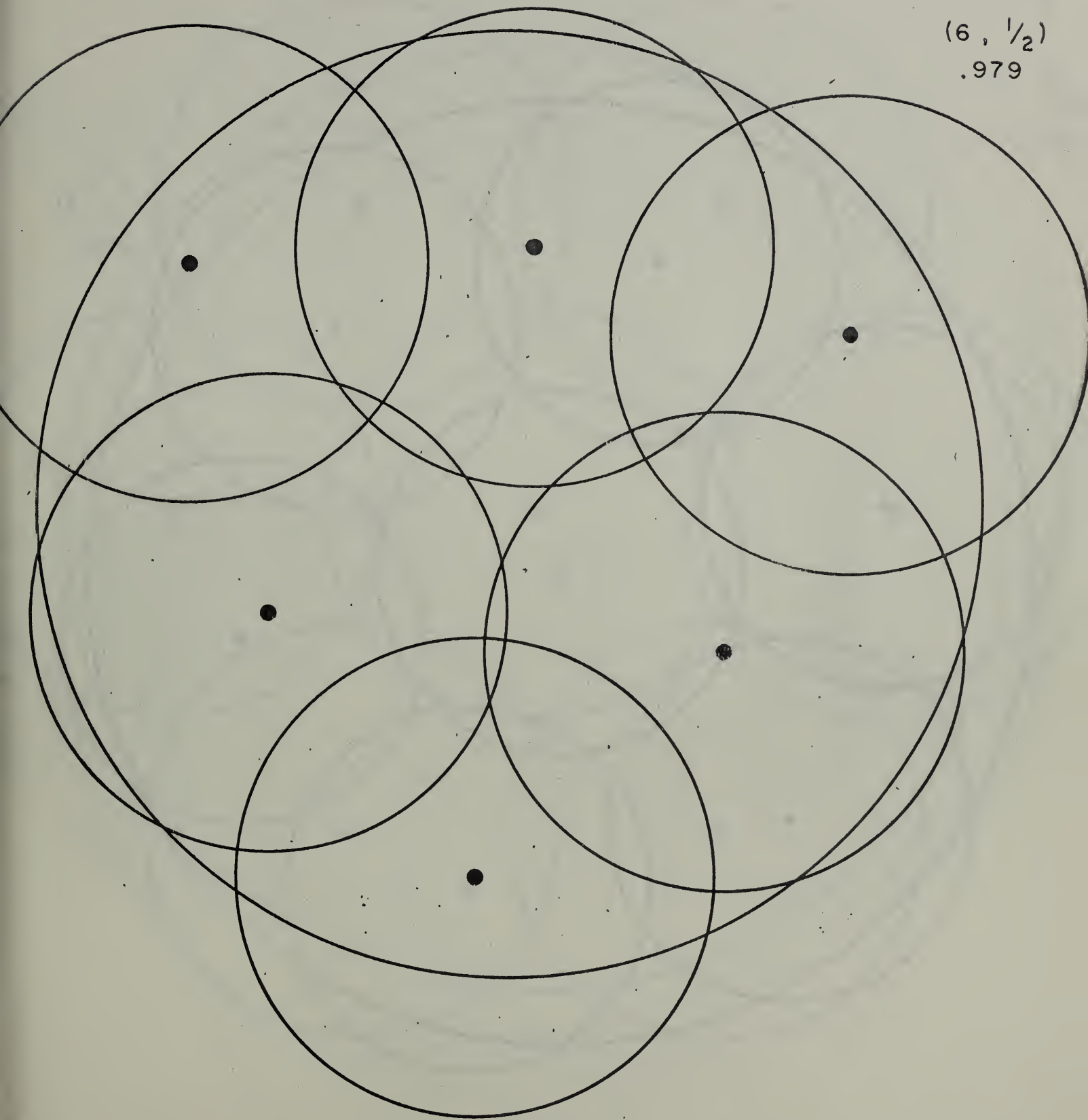
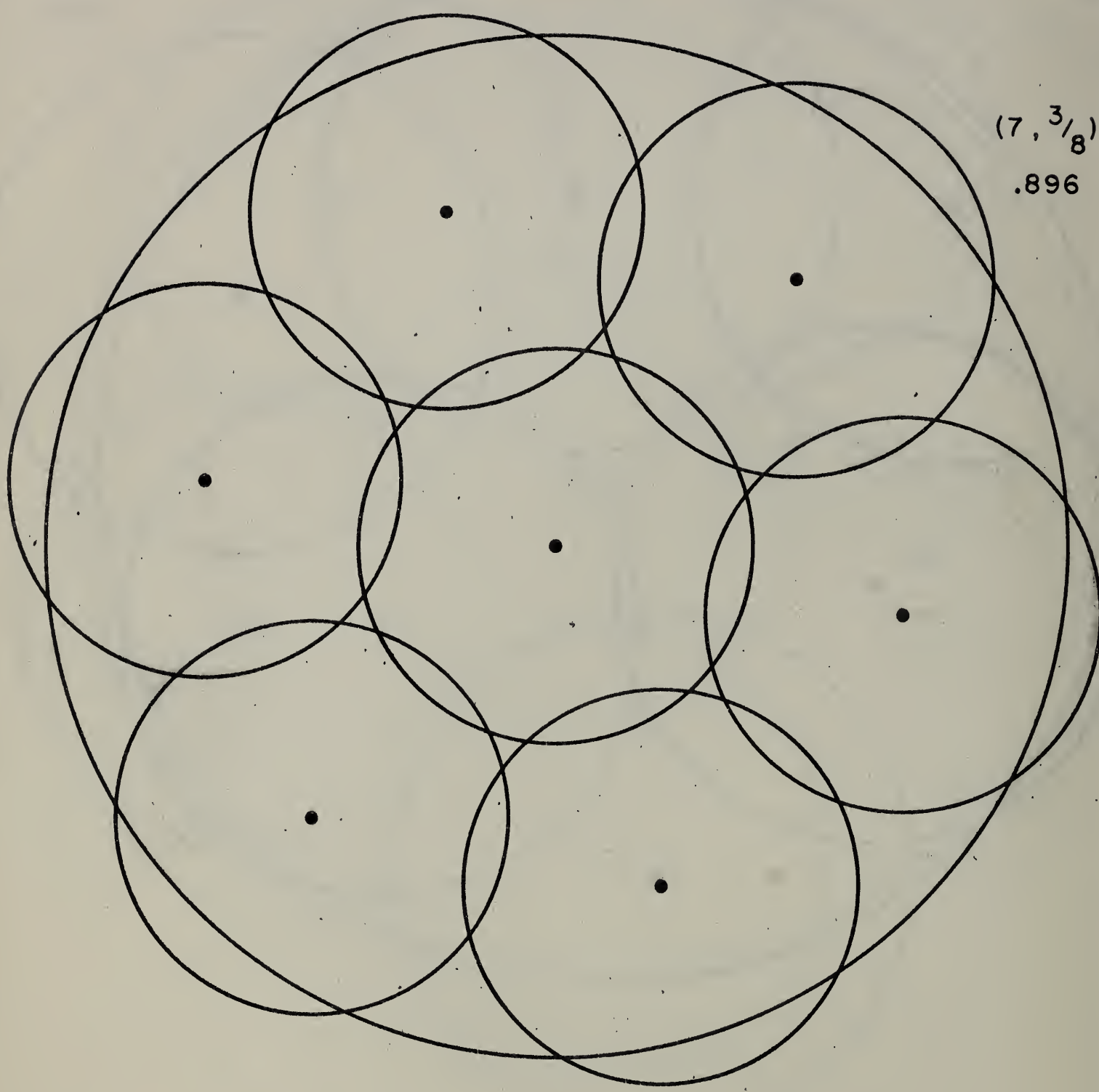


FIG.33



$(7, \frac{3}{8})$
.896

FIG. 34

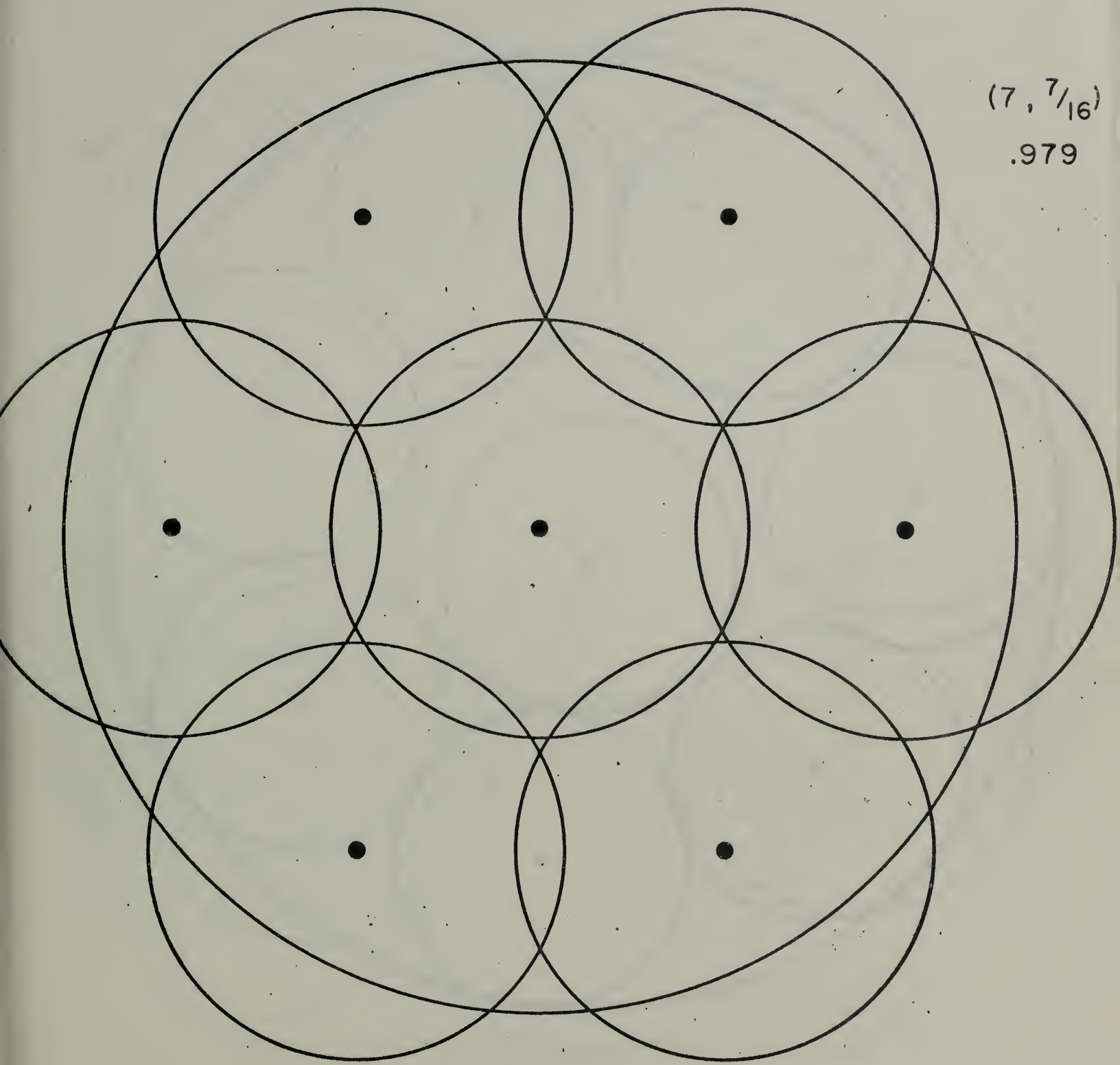


FIG.35

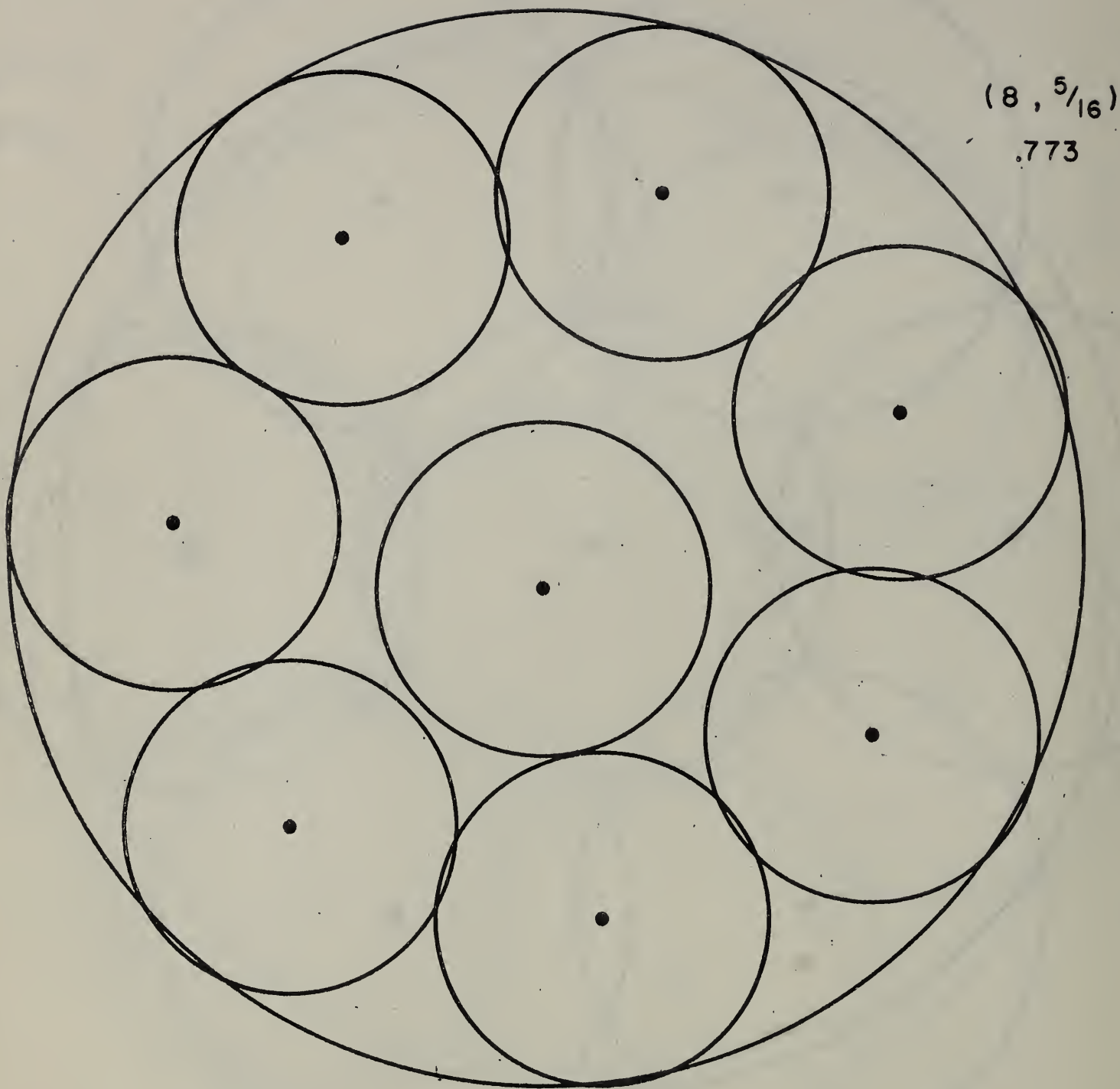
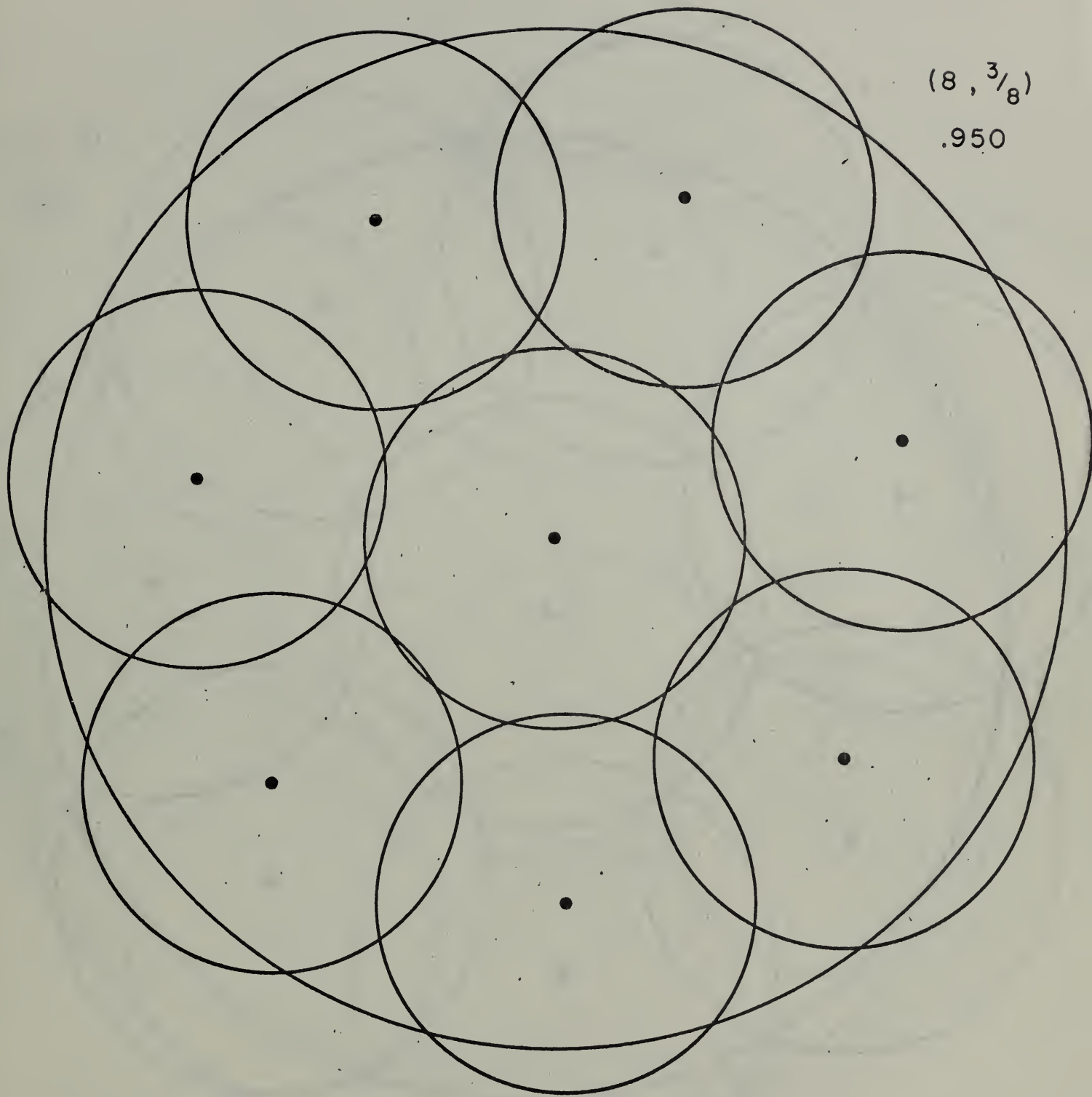


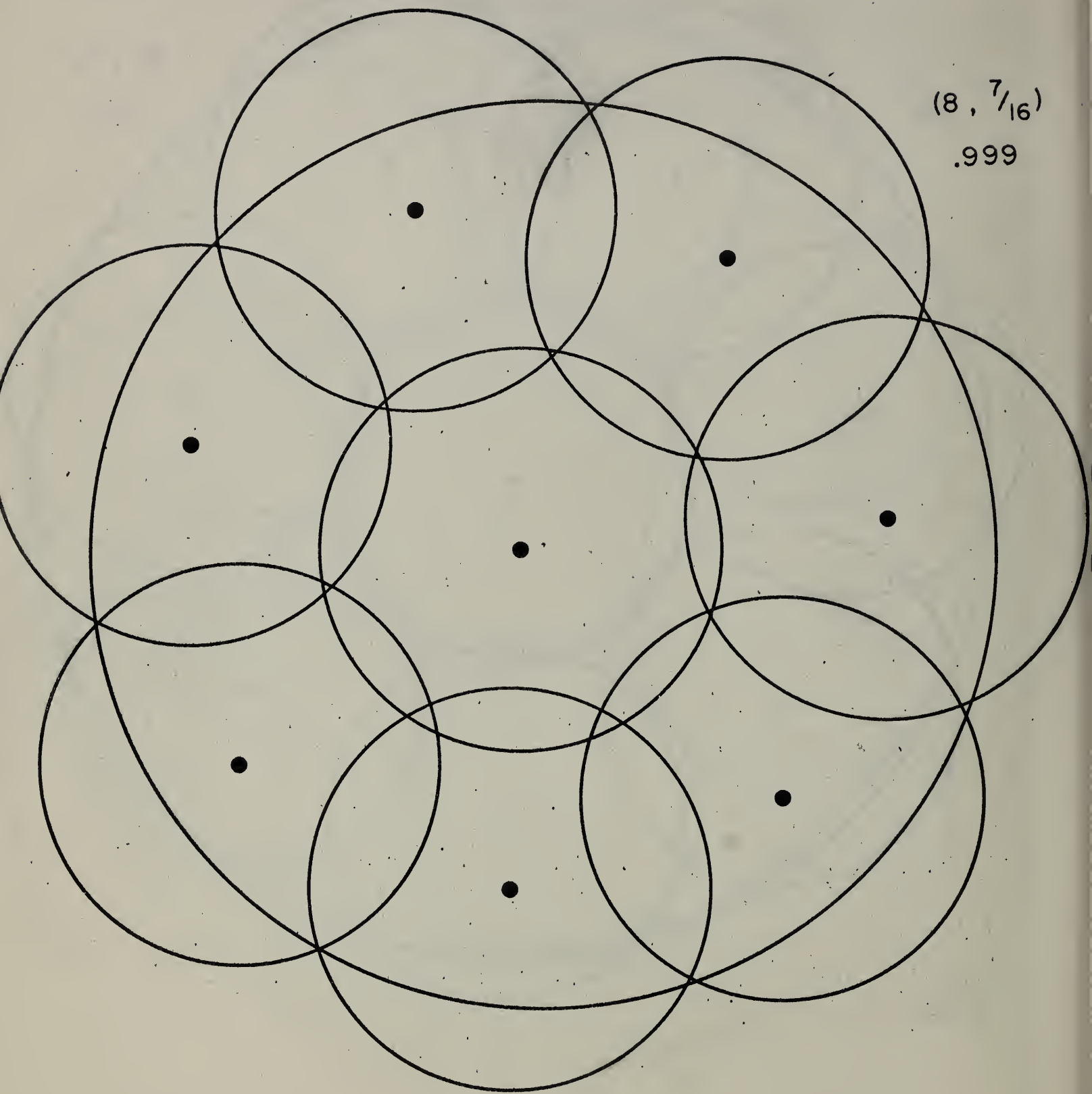
FIG. 36



$(8, \frac{3}{8})$

.950

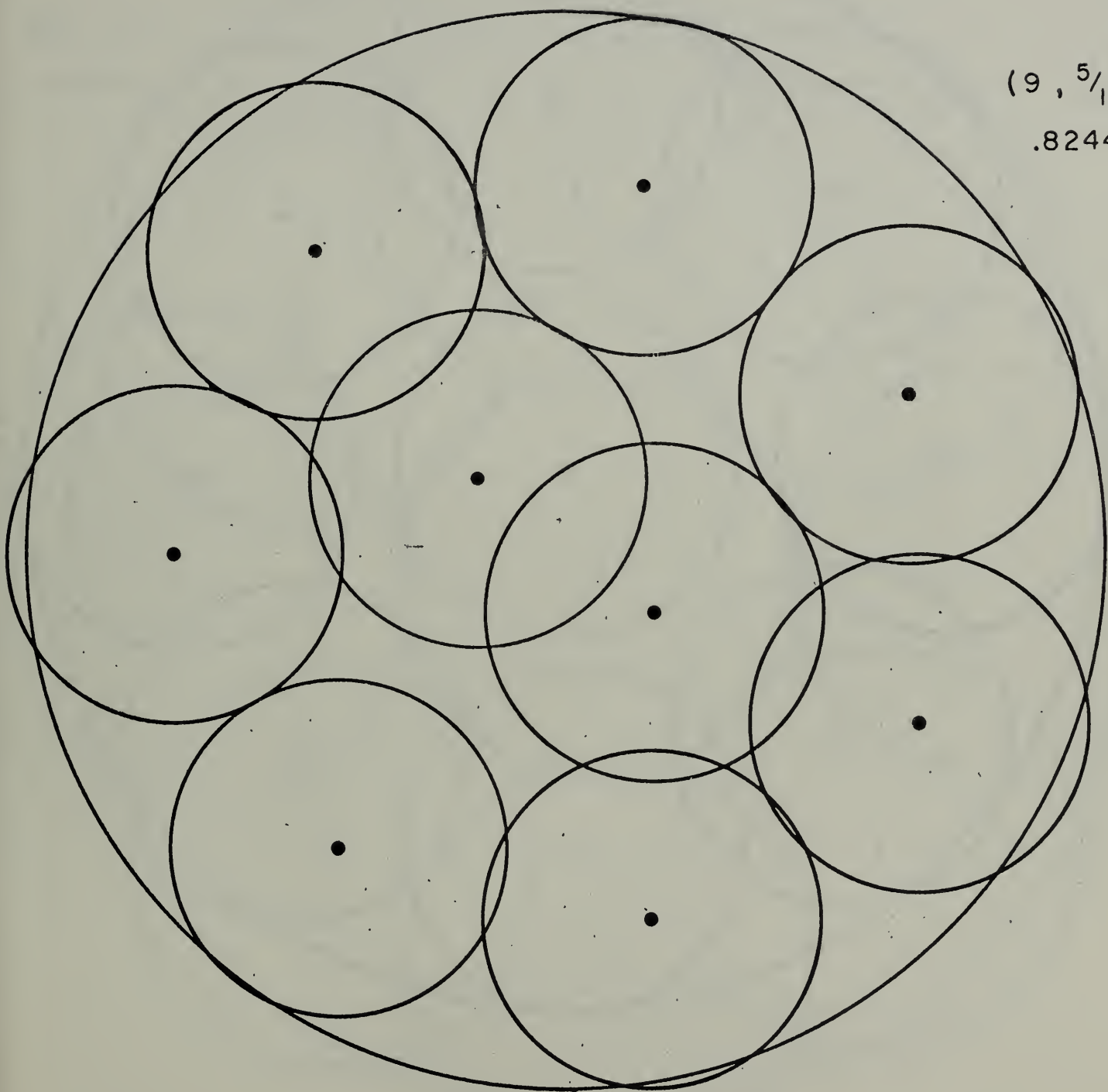
FIG. 37



$(8, \frac{7}{16})$

.999

FIG. 38



$(9, \frac{5}{16})$

.8244

FIG. 39

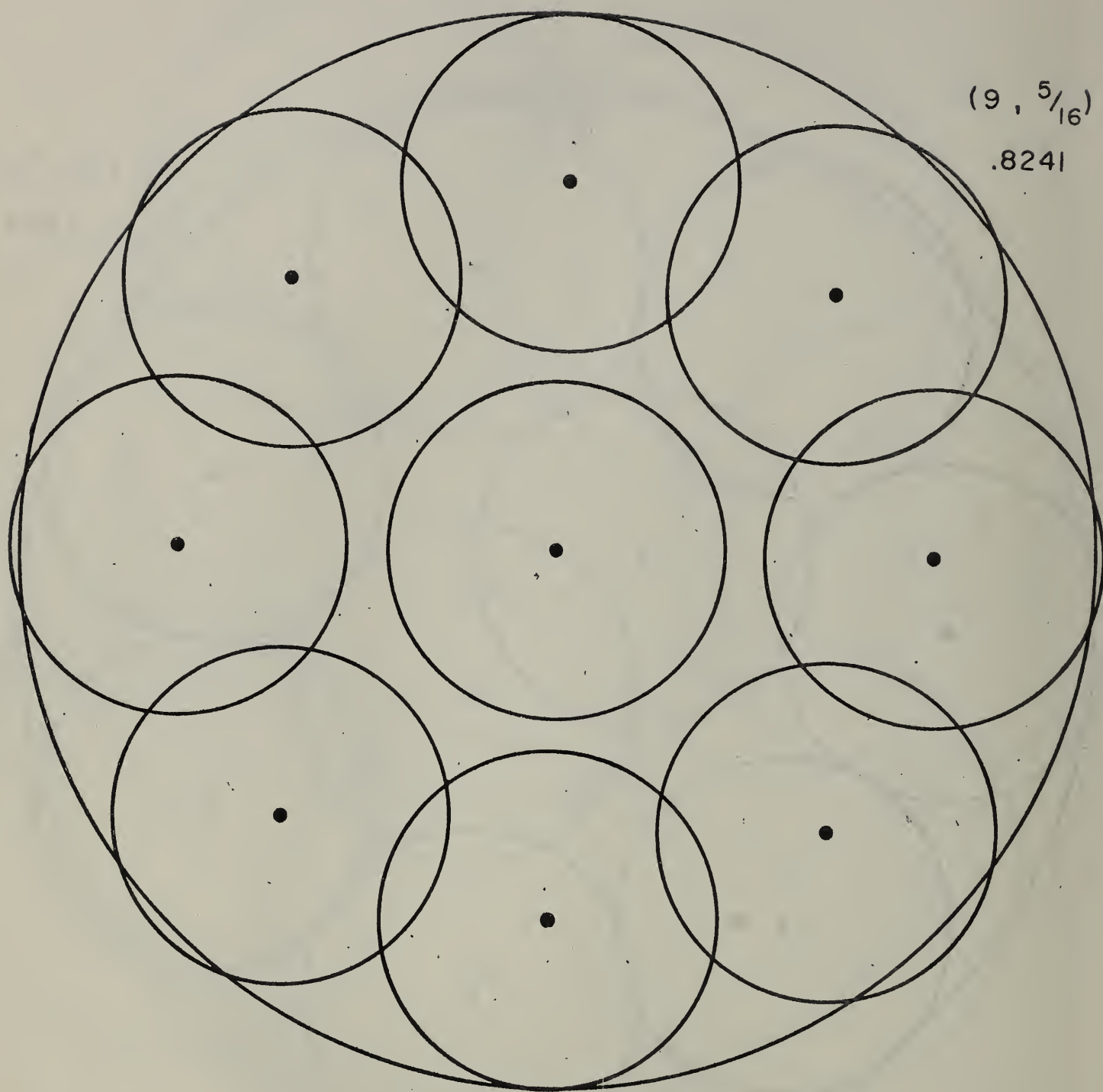


FIG. 40

$(9, \frac{3}{8})$
.978

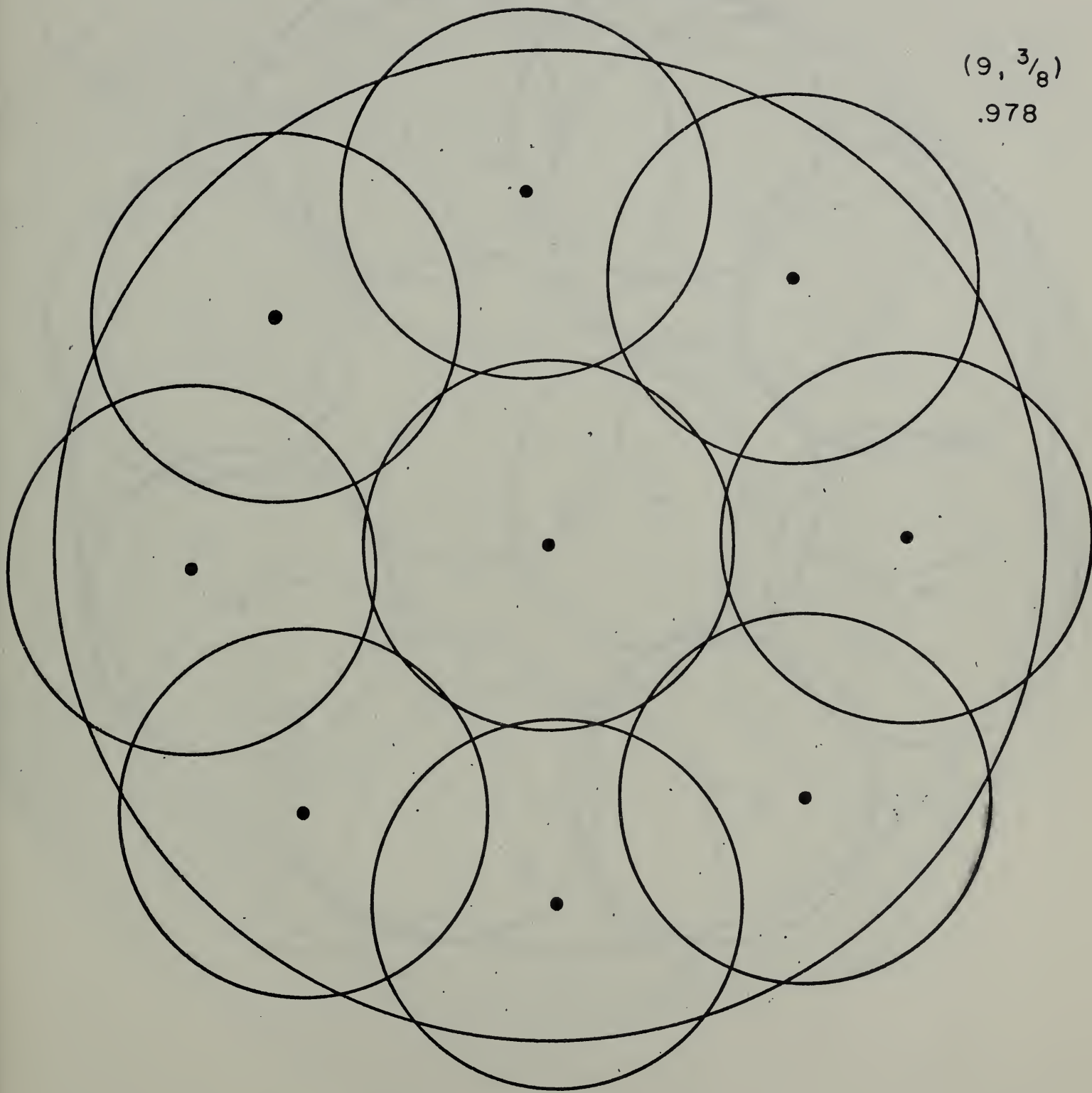


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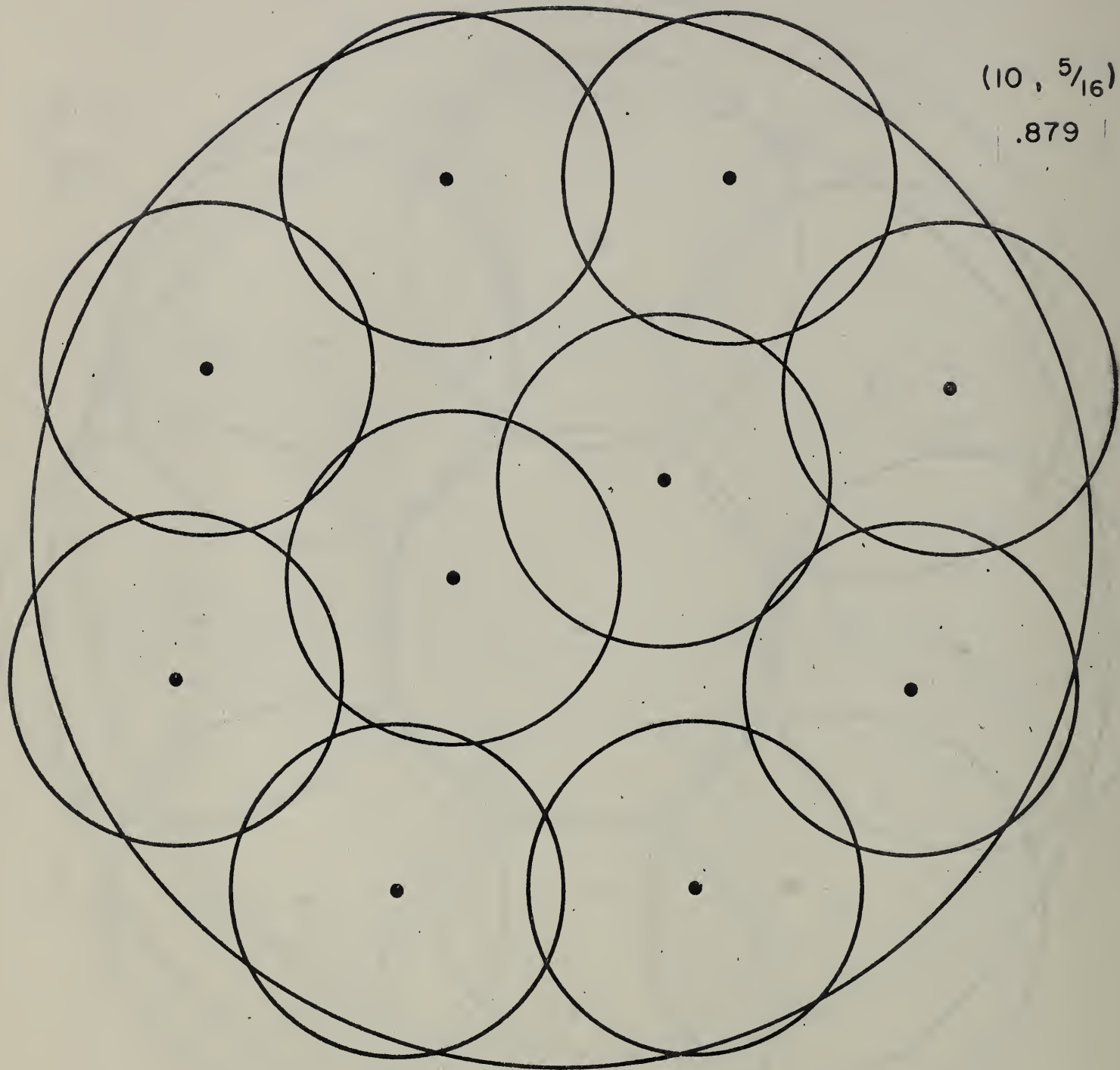


FIG. 42

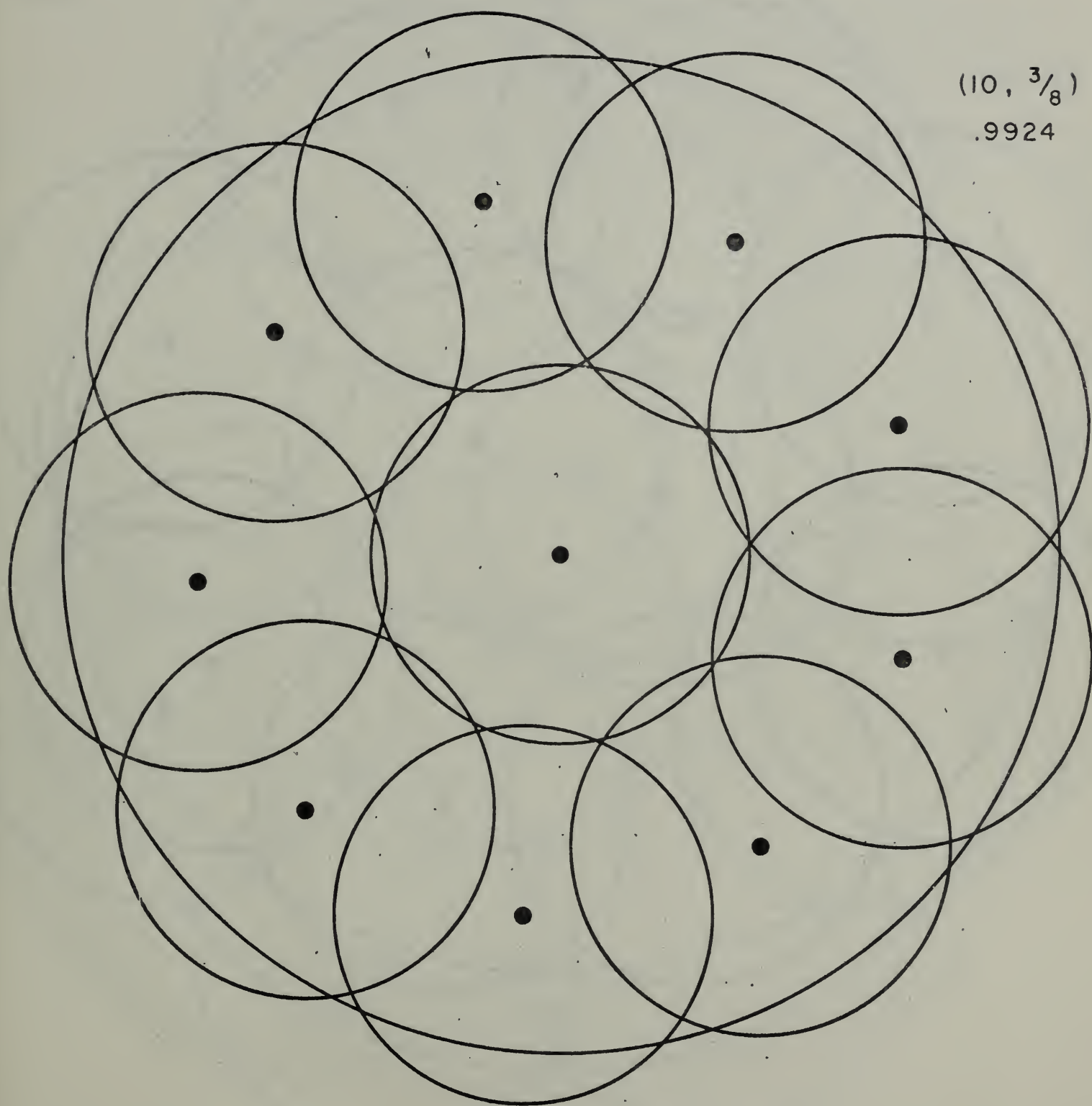


FIG. 43

$(10, \frac{3}{8})$
.9911

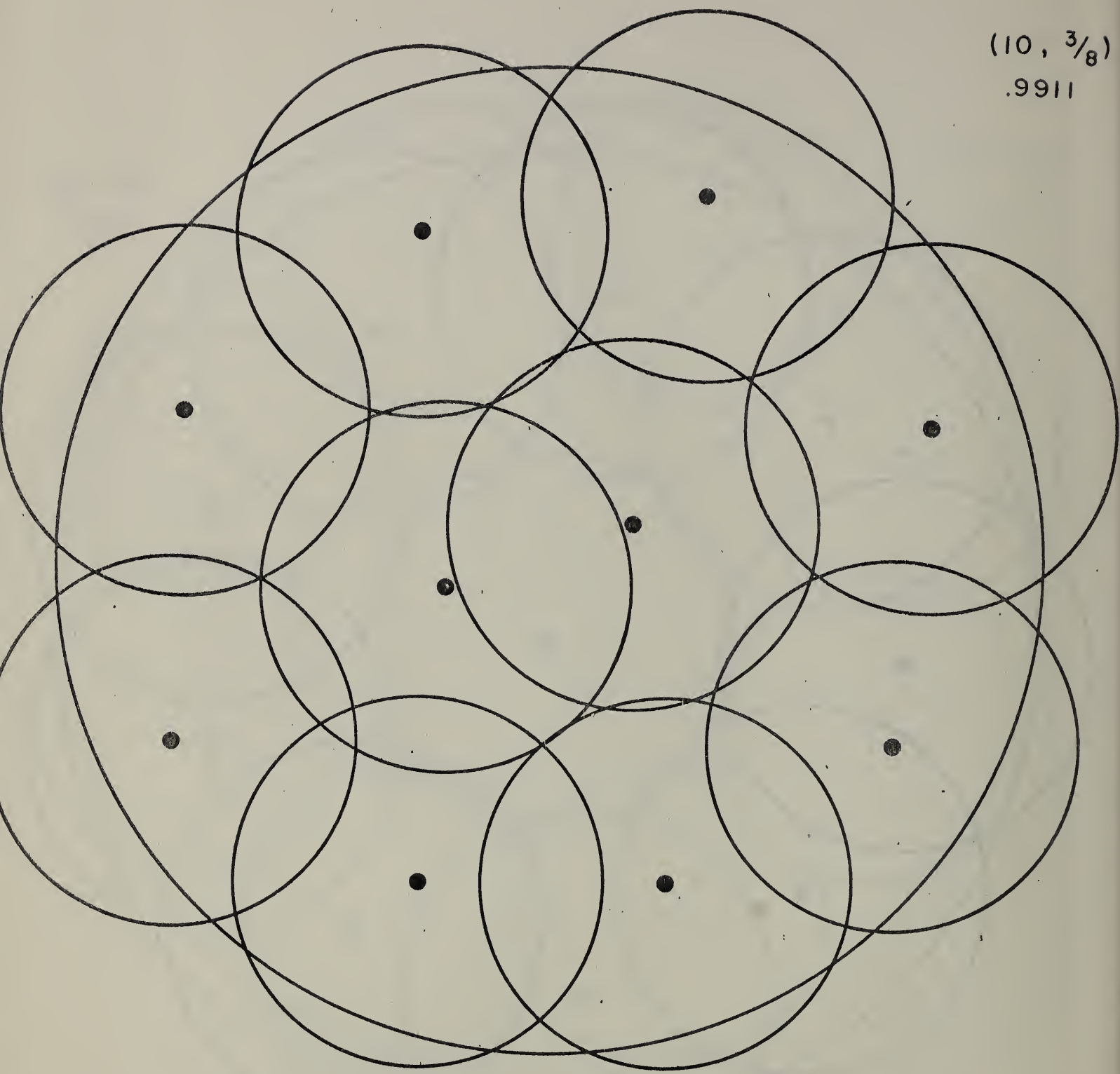


FIG. 44

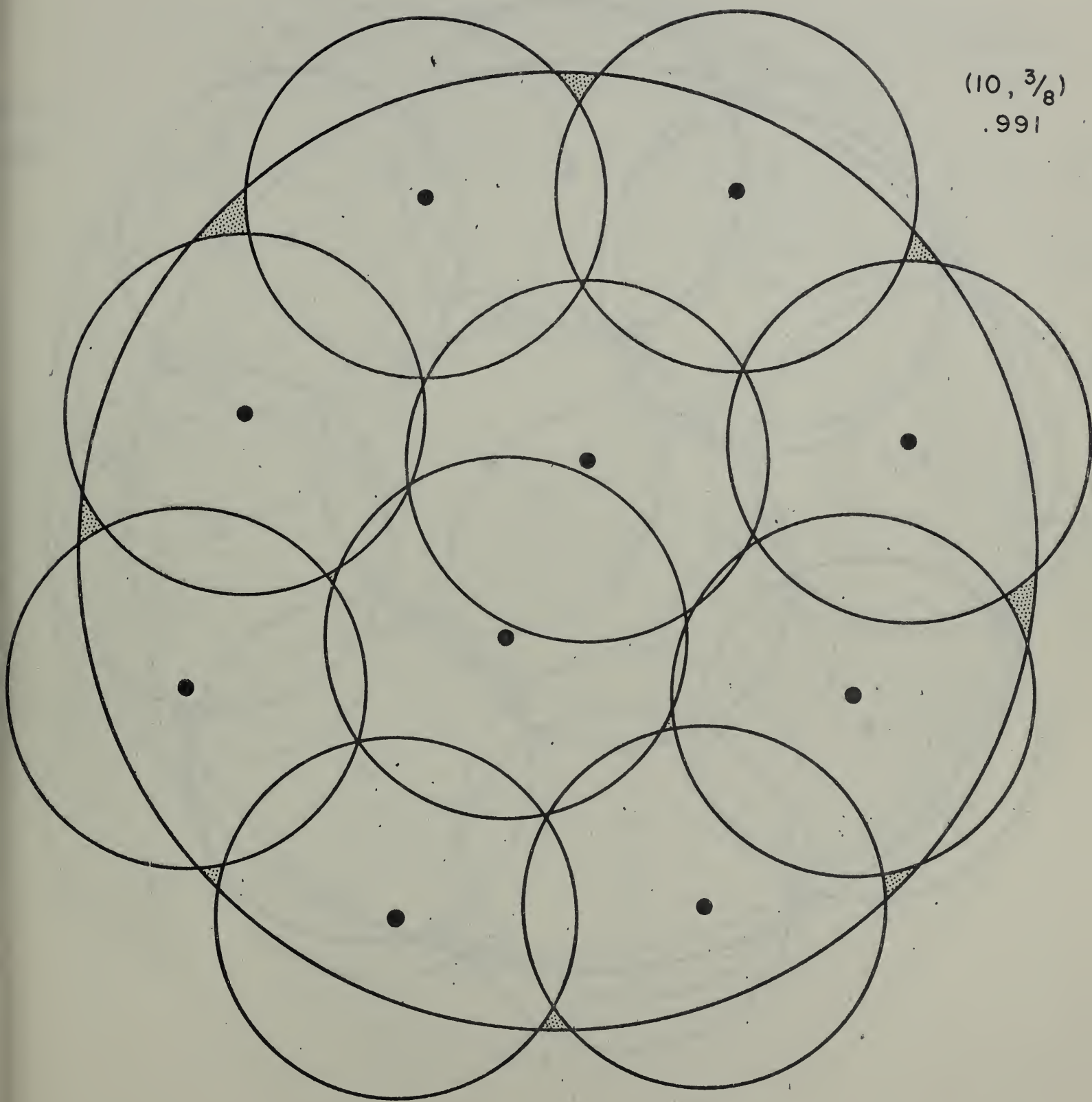
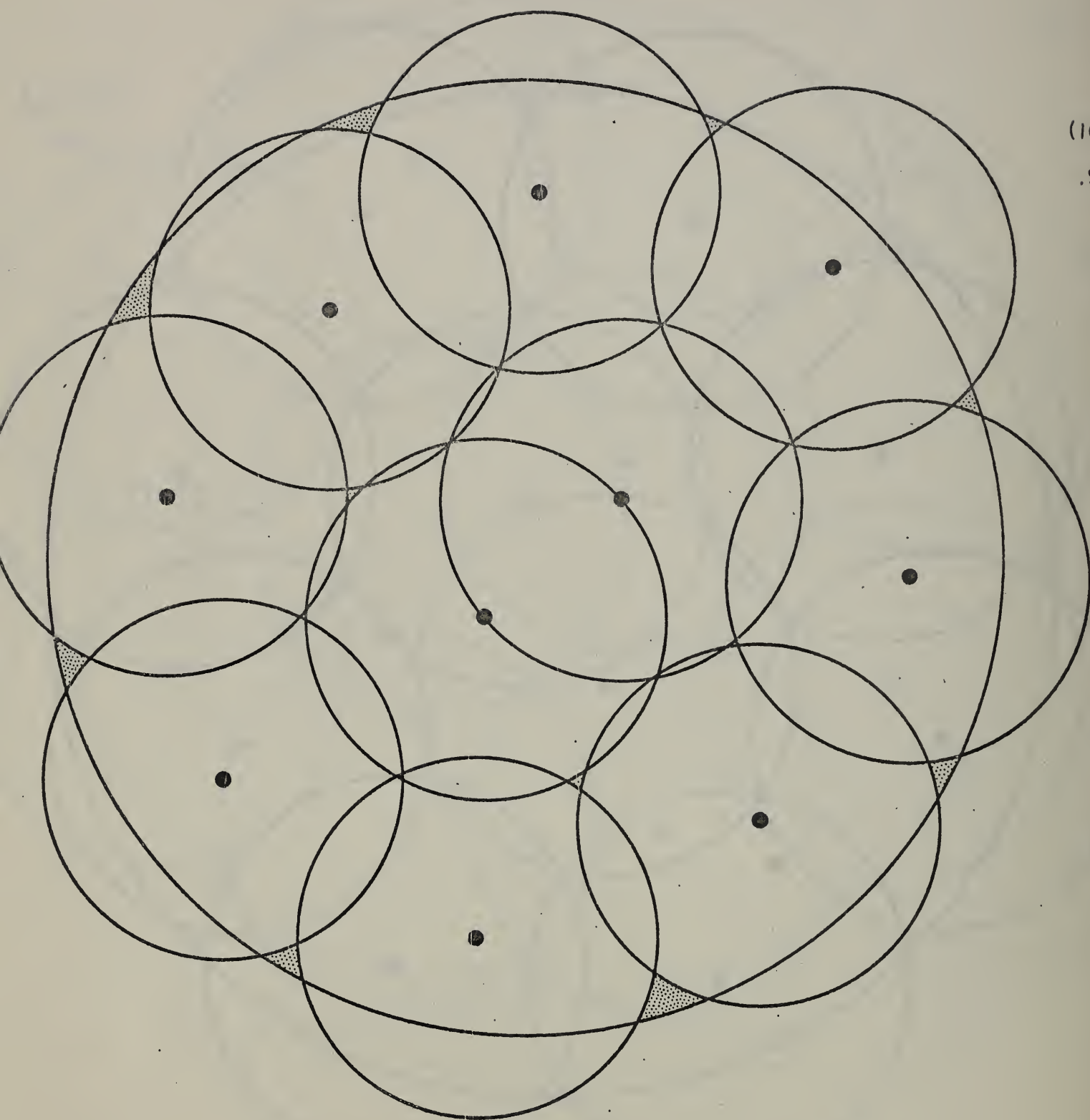
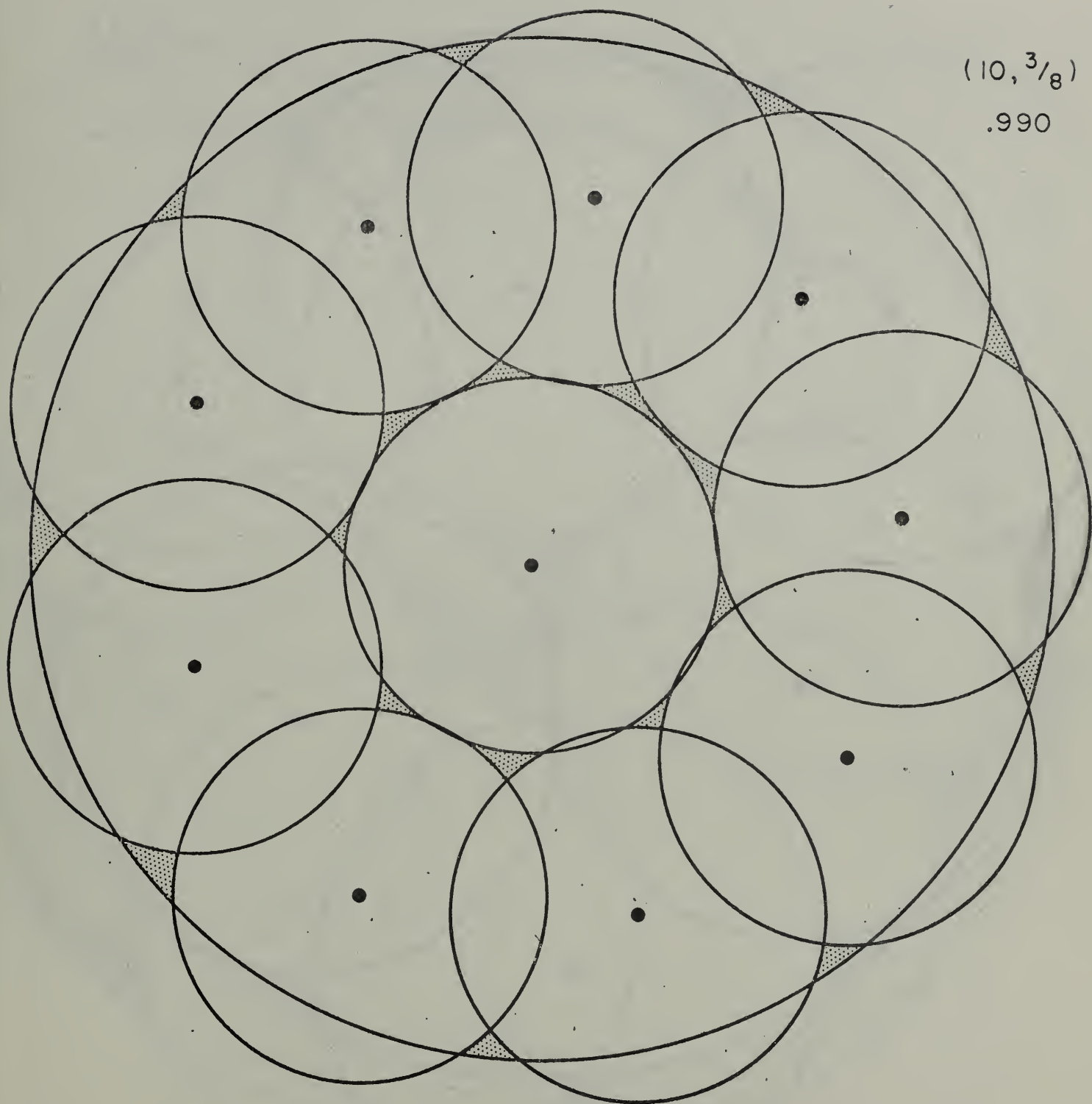


FIG.45



$(10, \frac{3}{8})$
.988

FIG. 46



$(10, \frac{3}{8})$
.990

FIG. 47

$(6, \frac{1}{2})$
.979

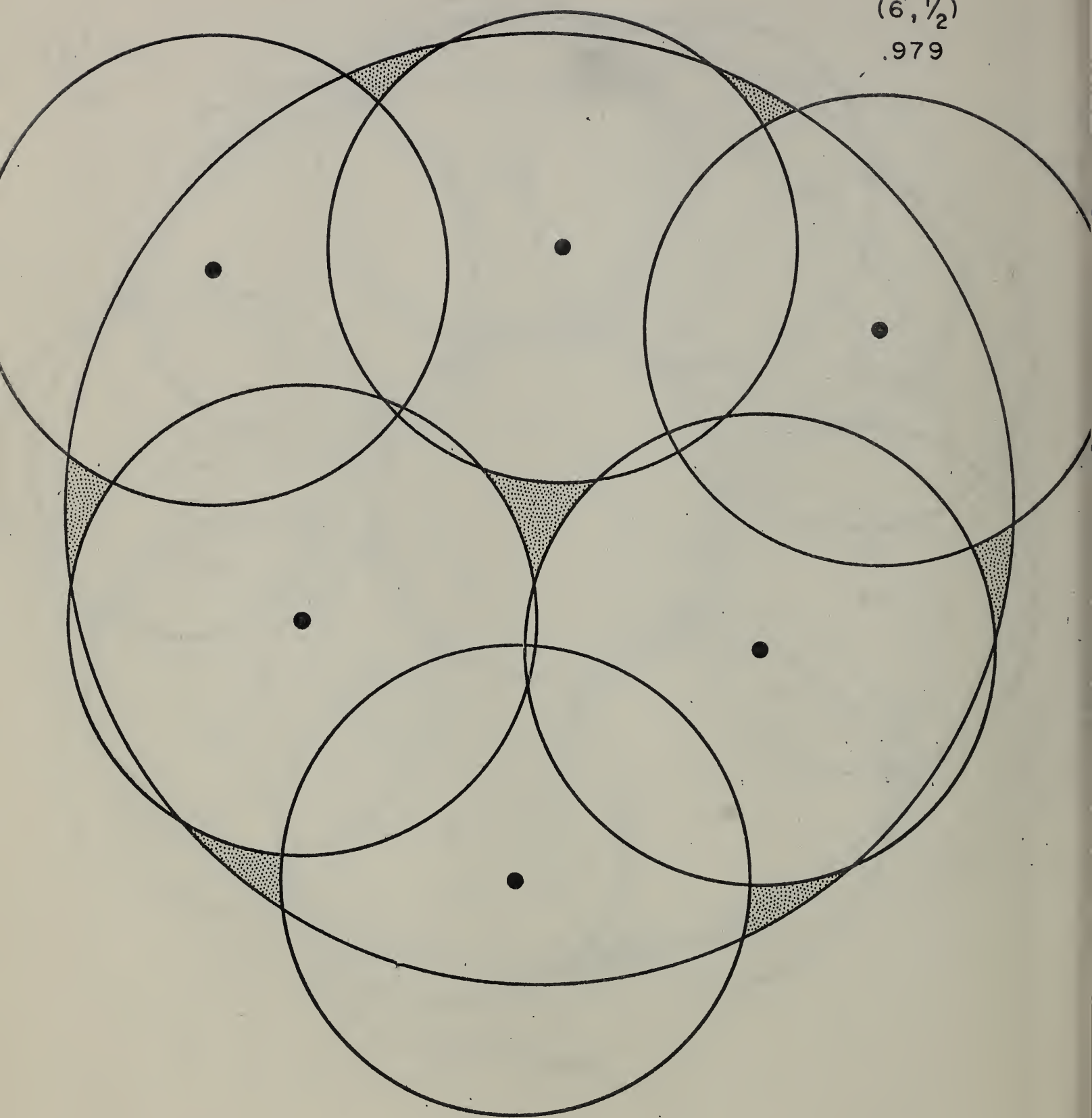


FIG.48

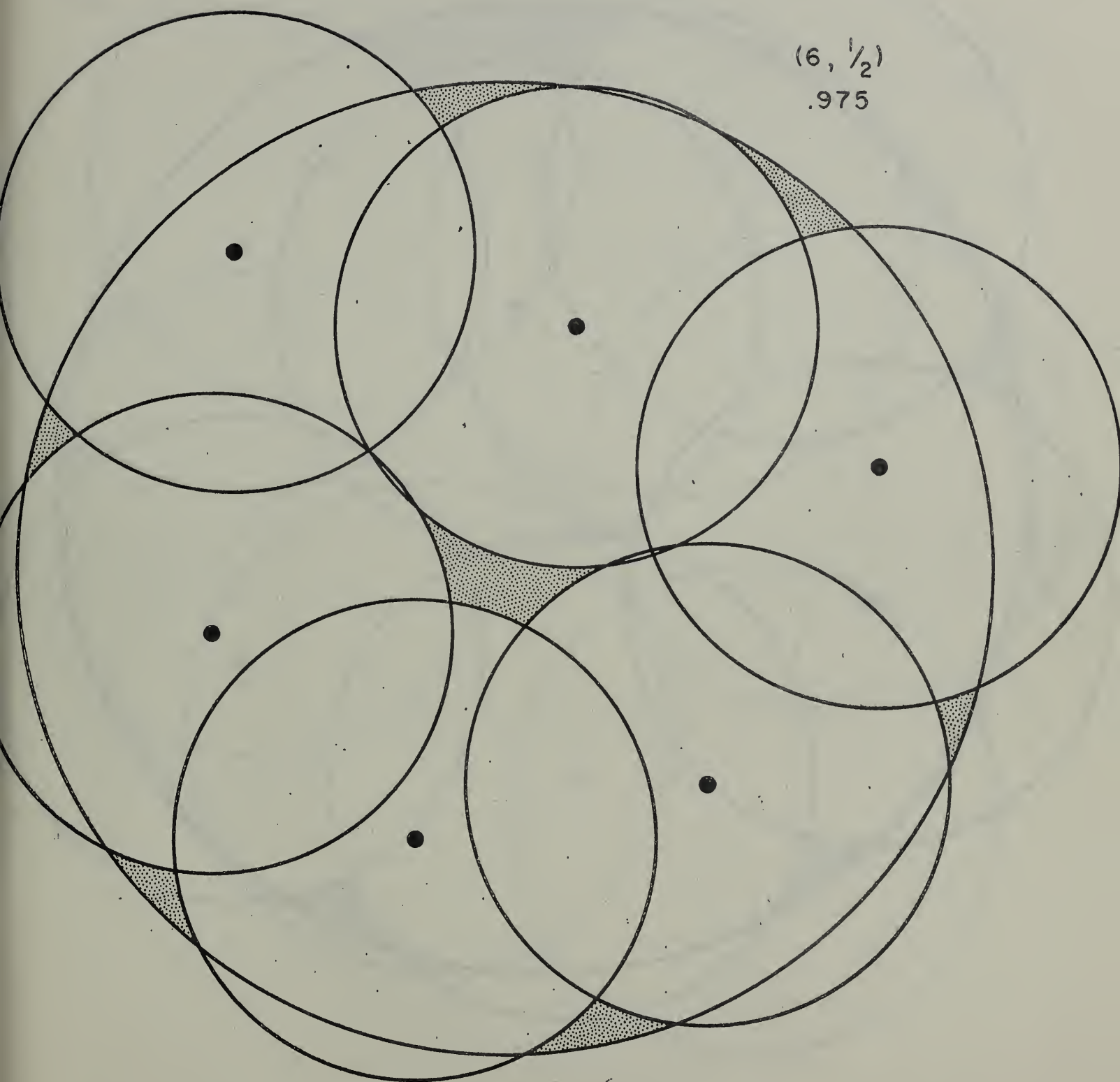


FIG.49

$(6, \frac{1}{2})$
.967

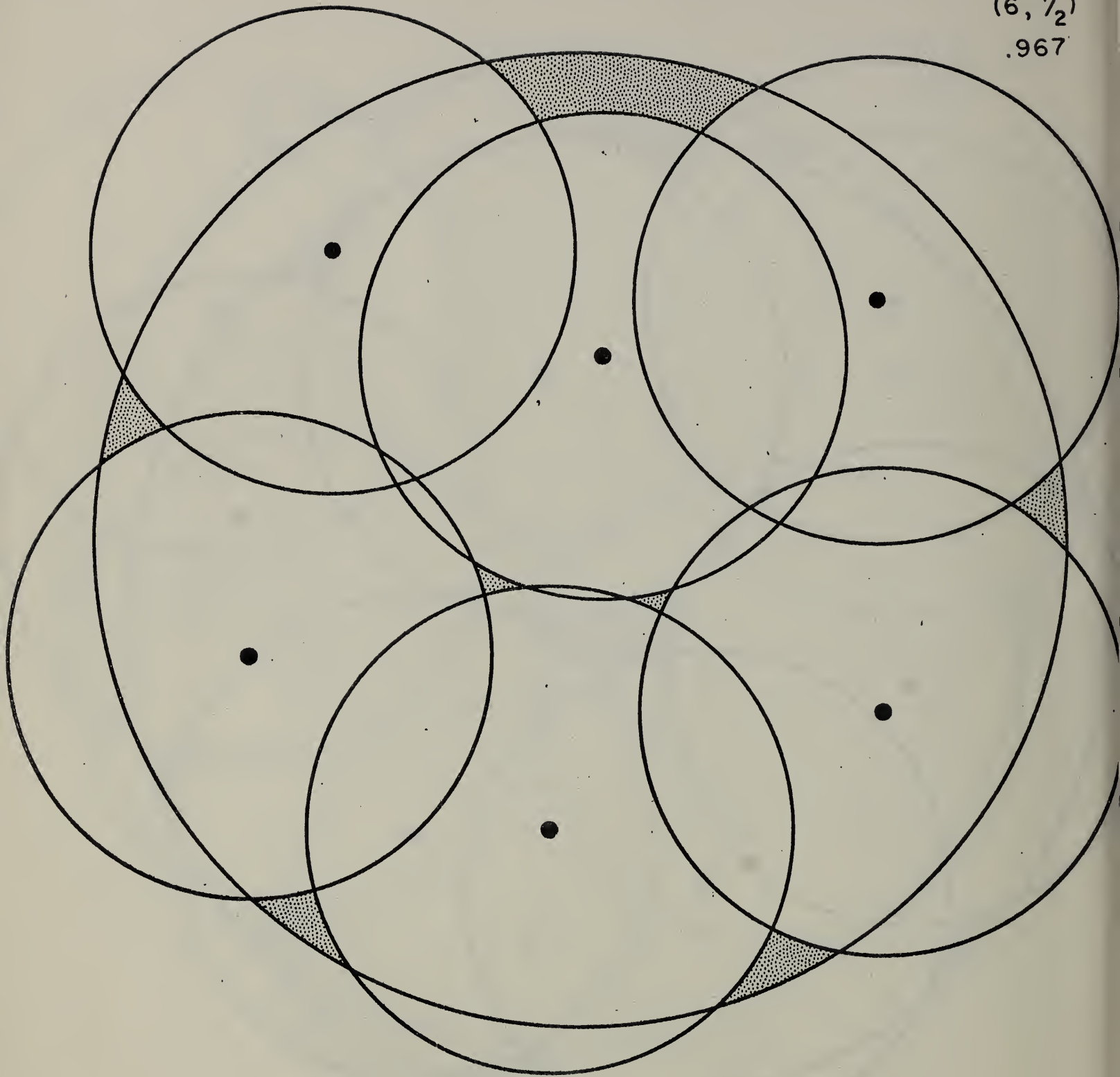


FIG.50

$(6, \frac{1}{2})$

.974

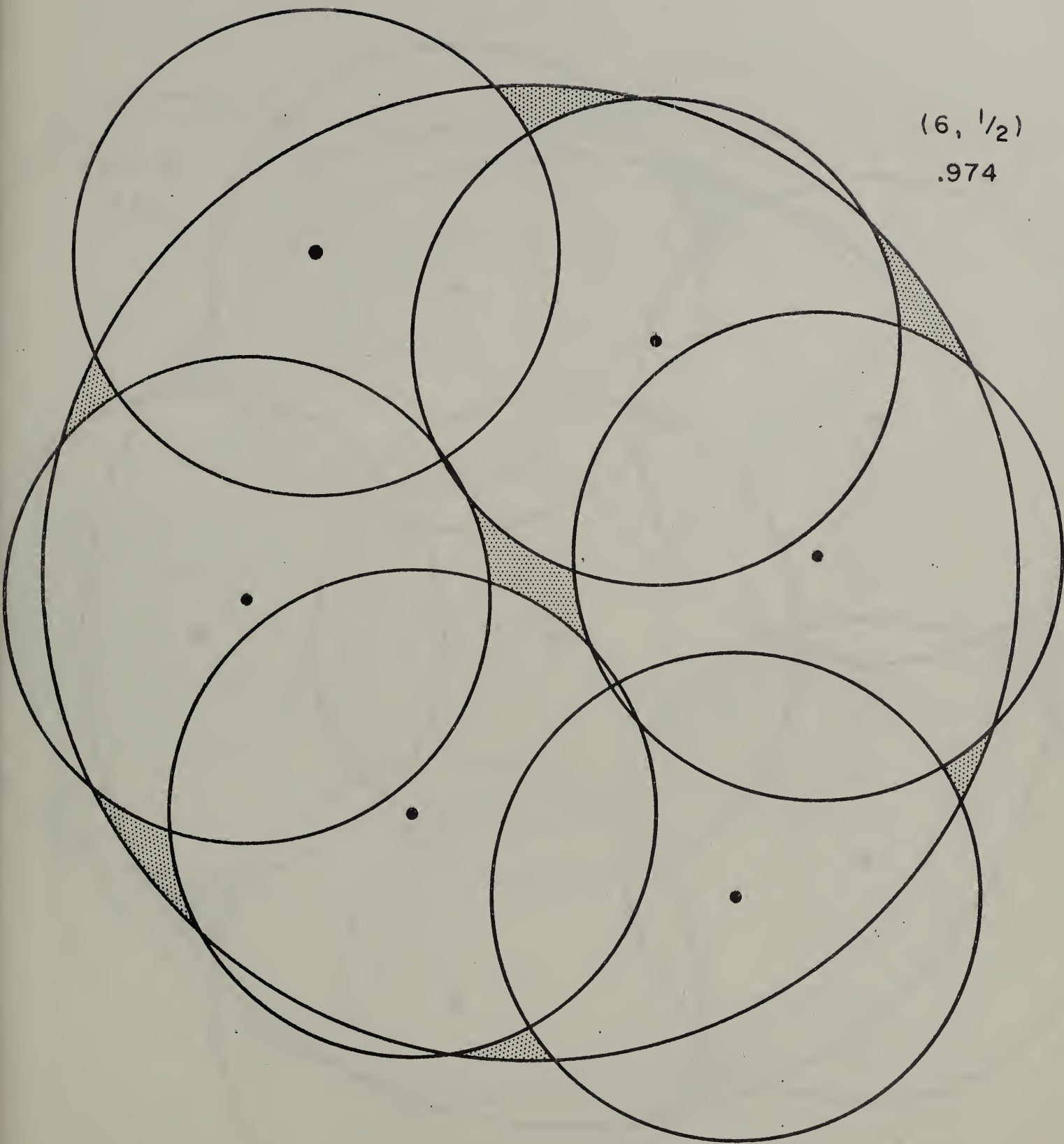


FIG. 51

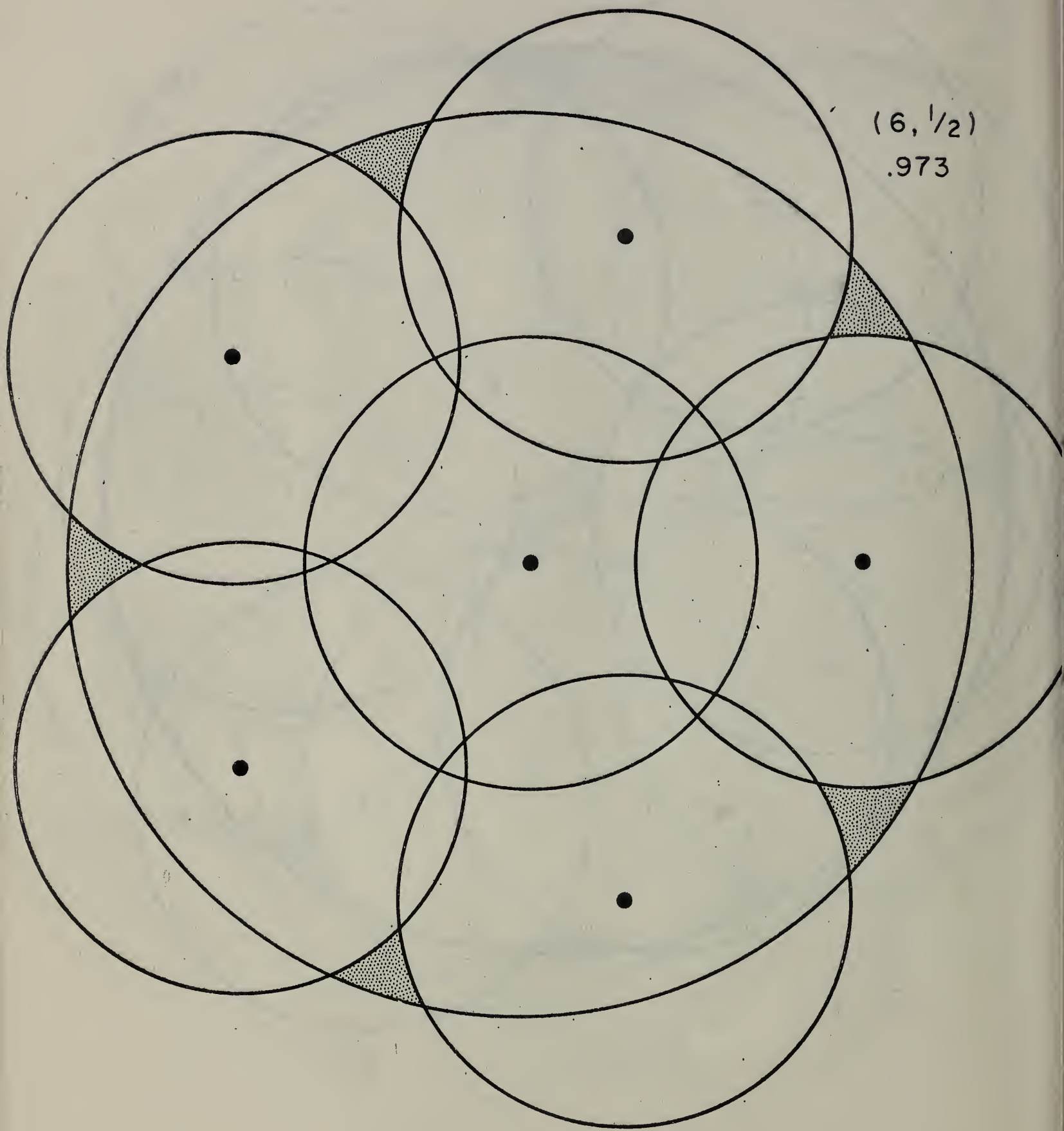
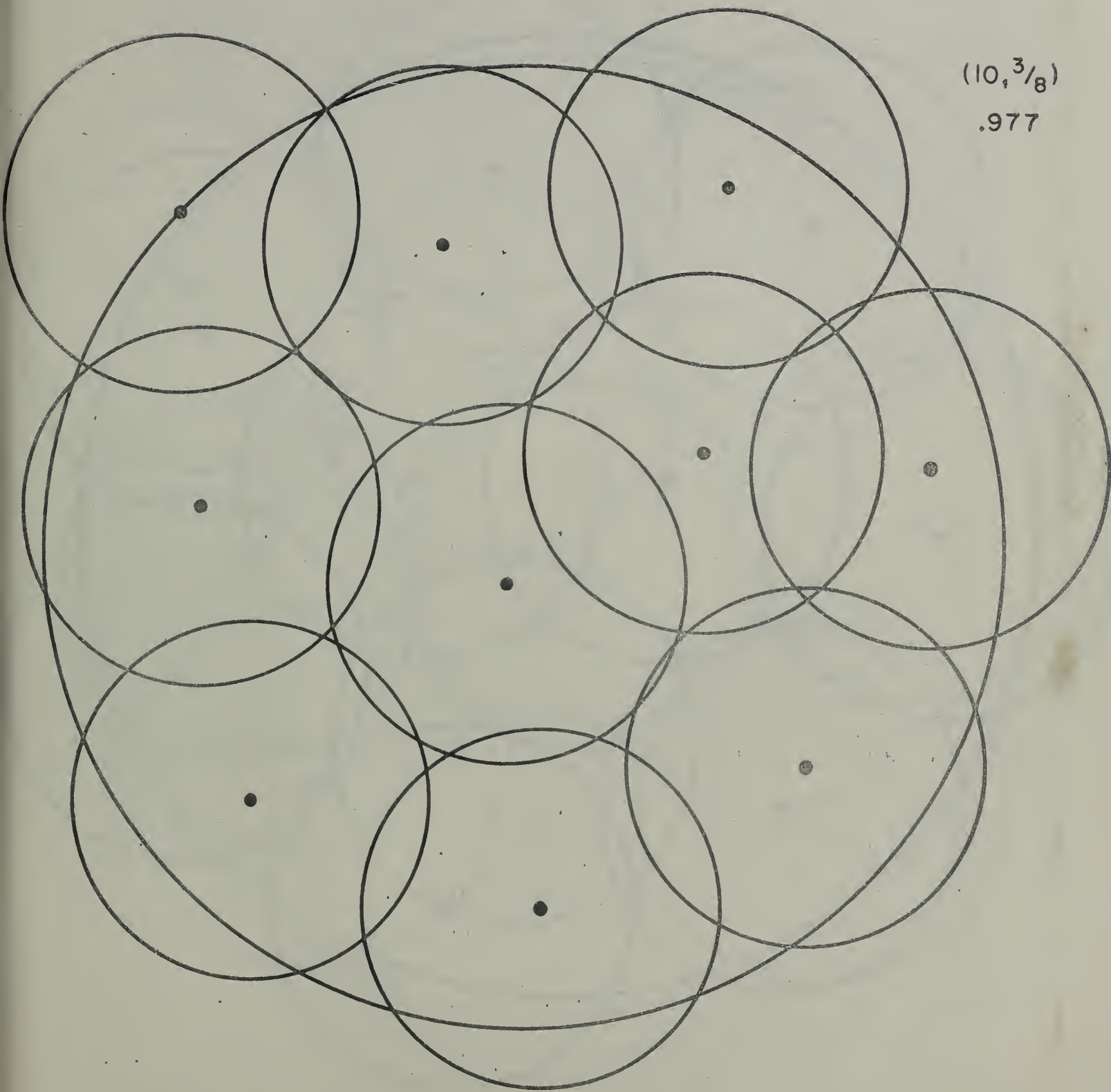


FIG. 52



$(10, \frac{3}{8})$

.977

FIG. 53

$(10, \frac{3}{8})$

.972

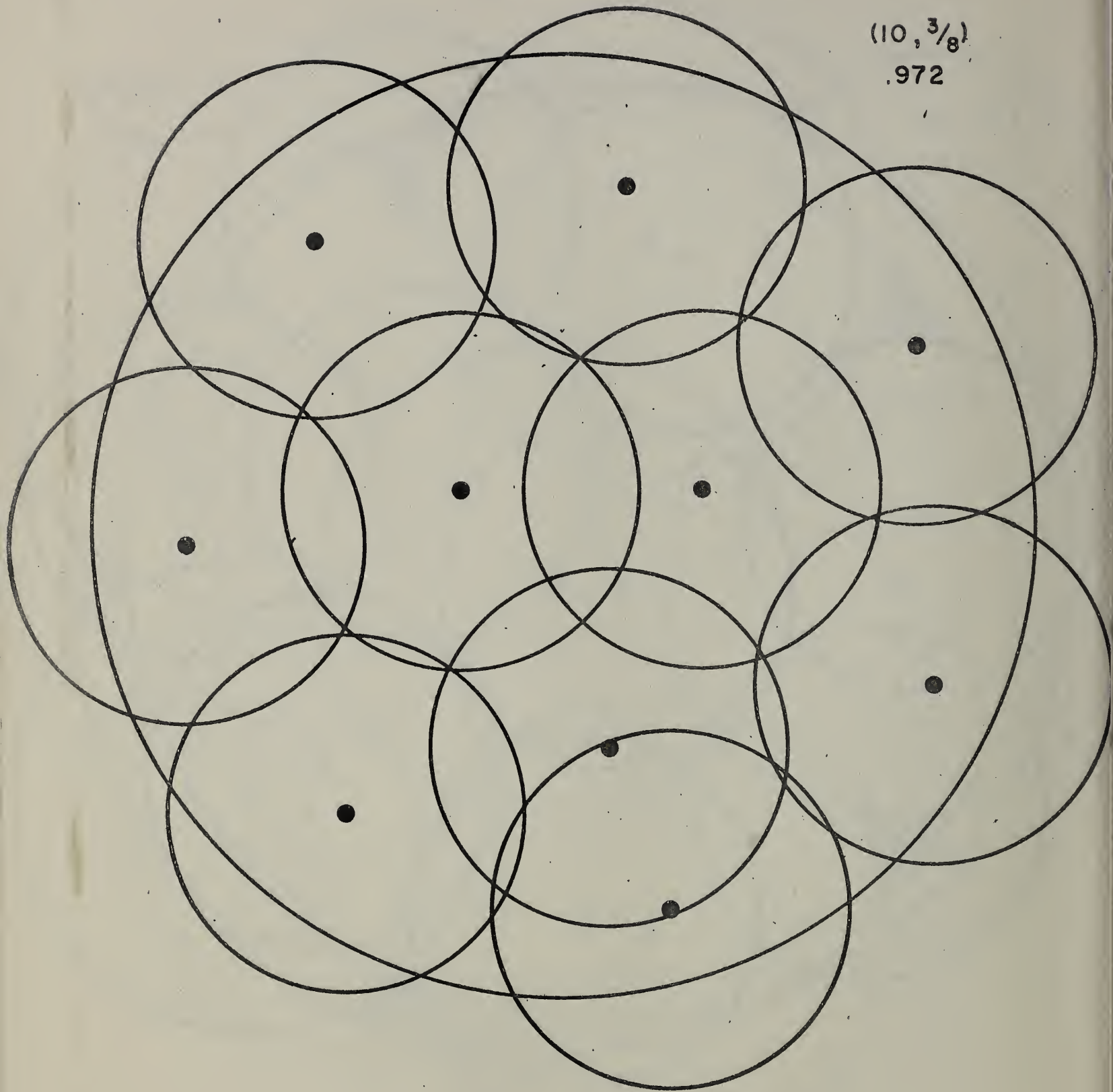


FIG. 54

$(10, \frac{3}{8})$
.985

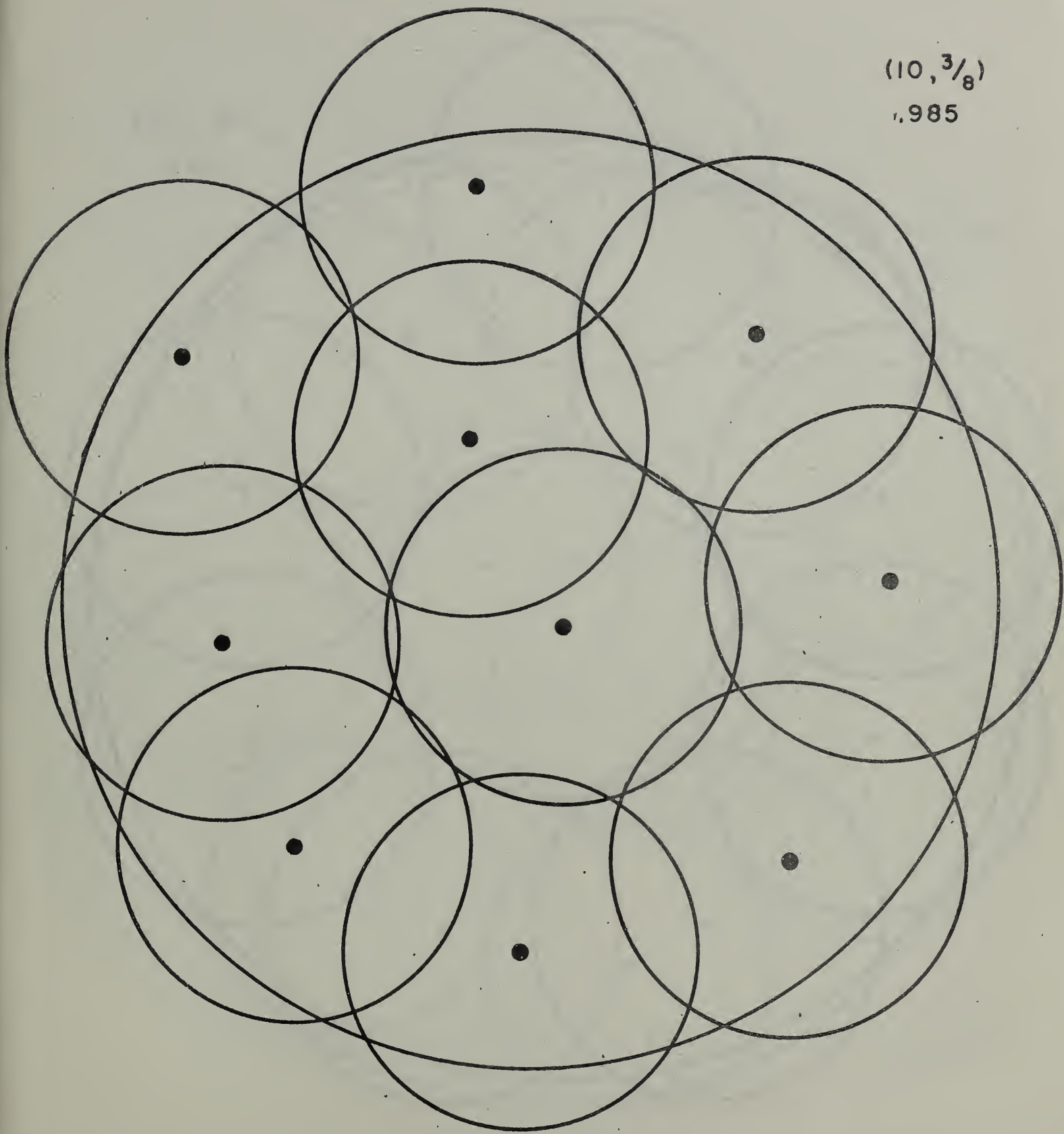
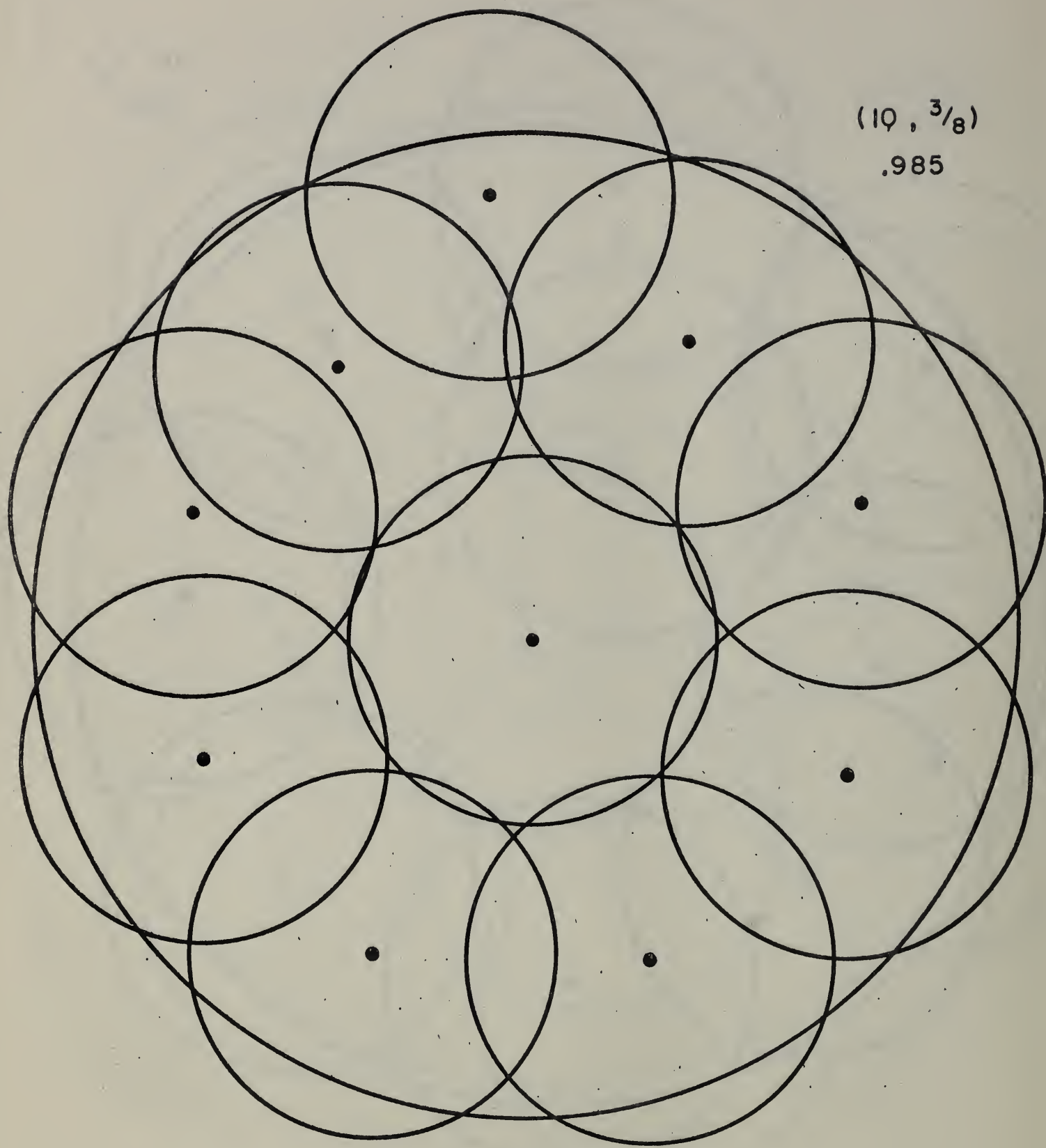


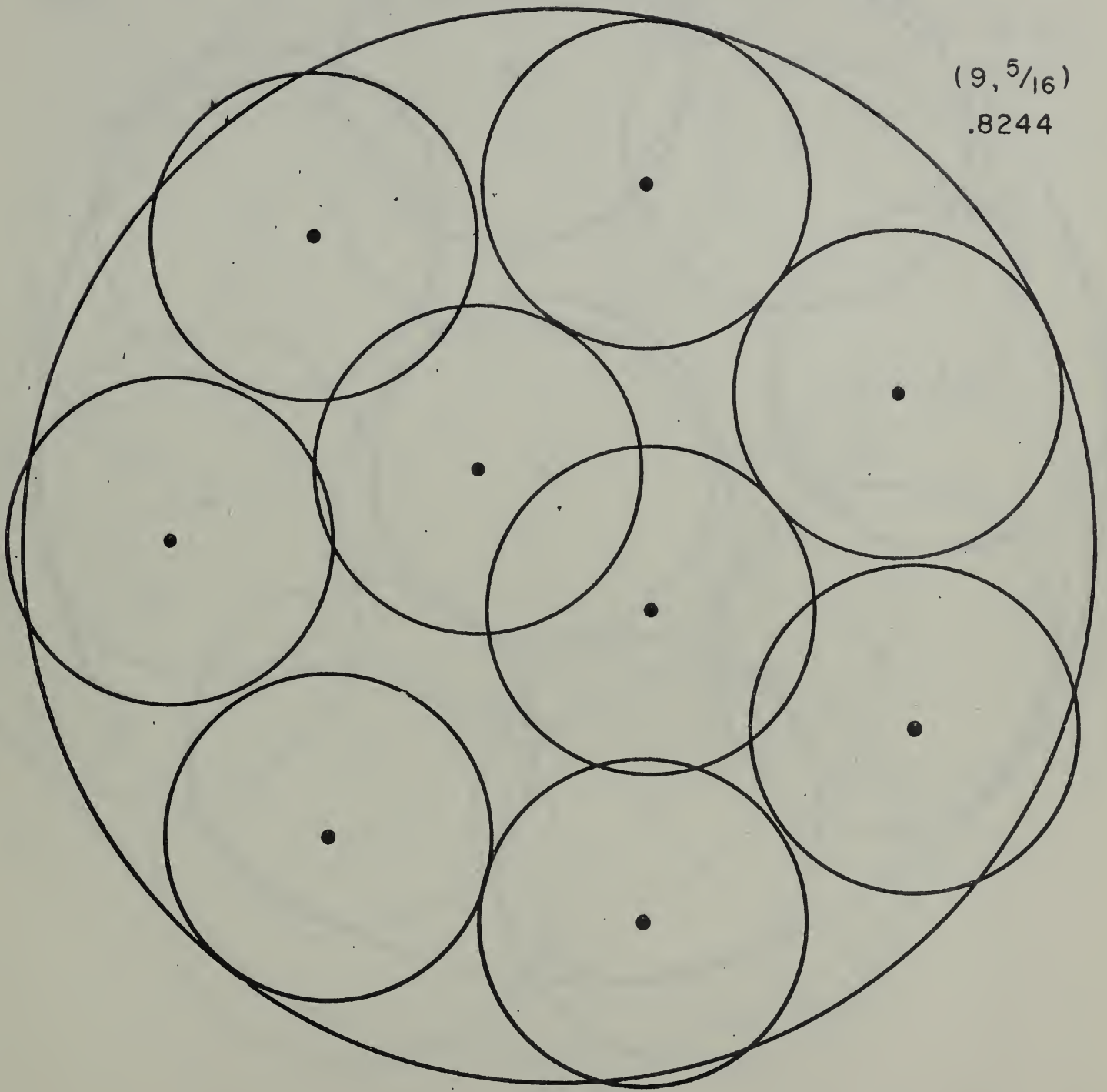
FIG. 55



$(10, \frac{3}{8})$

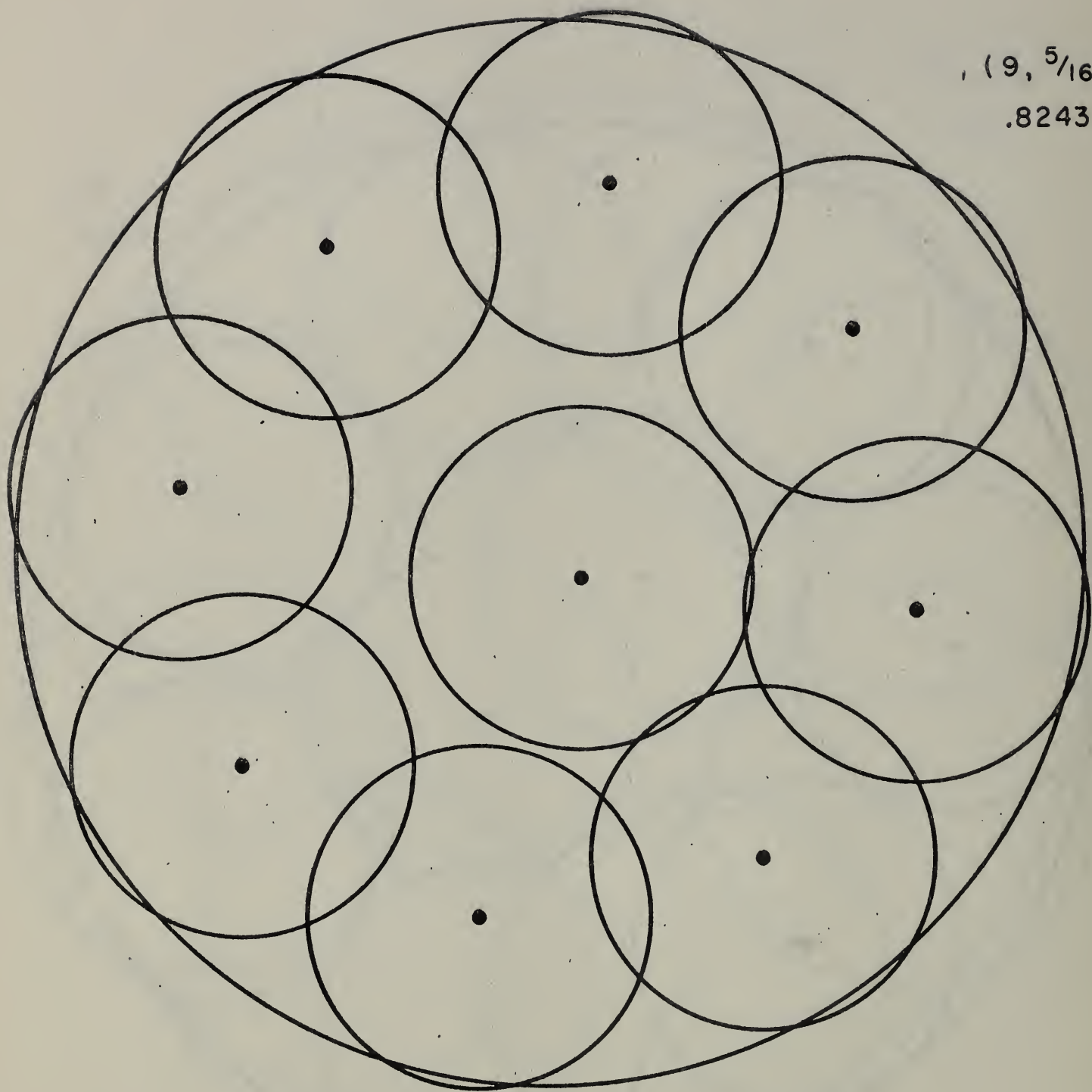
.985

FIG. 56



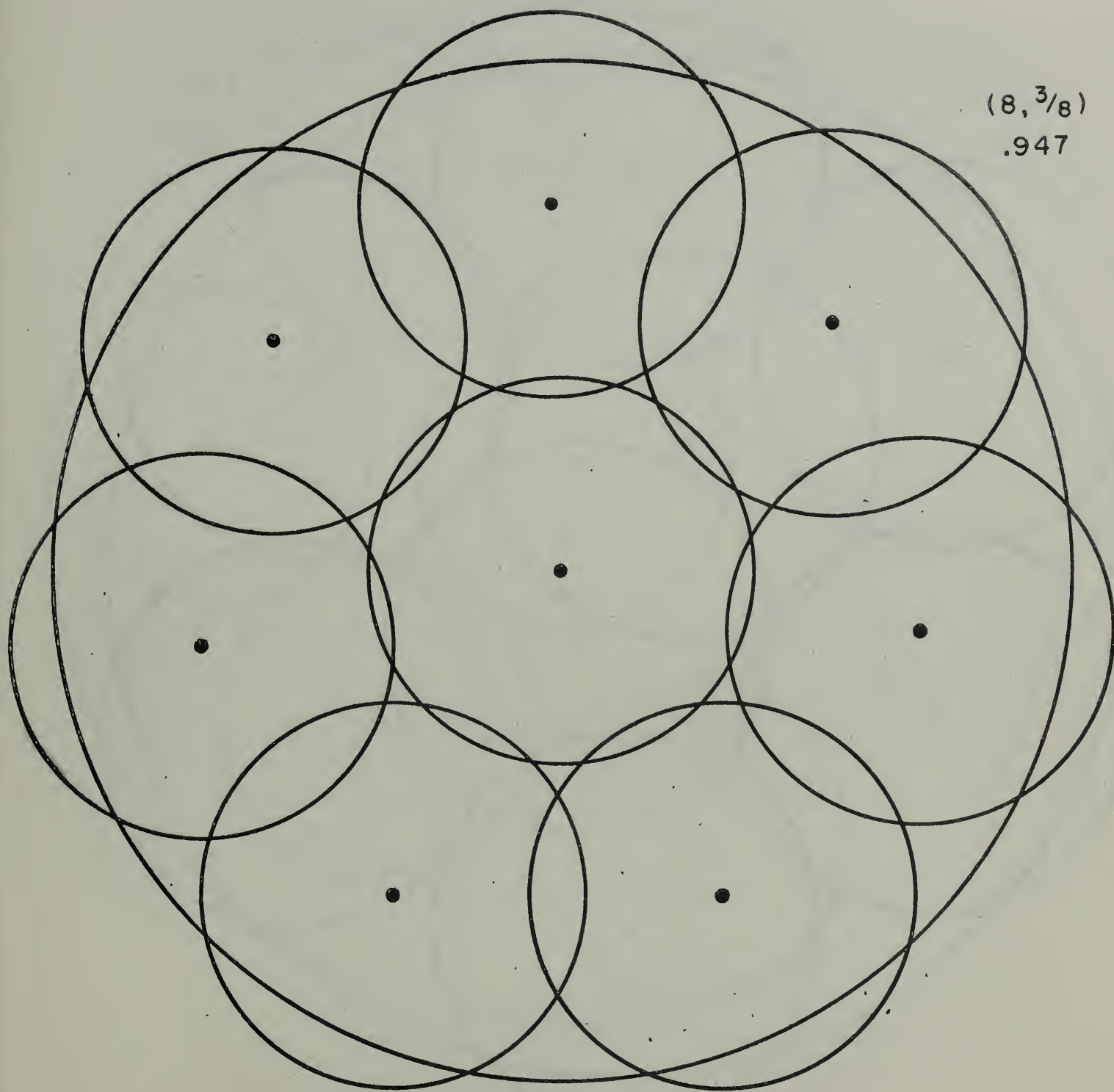
$(9, \frac{5}{16})$
.8244

FIG. 57



, $(9, \frac{5}{16})$
.8243

FIG. 58



$(8, \frac{3}{8})$

.947

FIG. 59

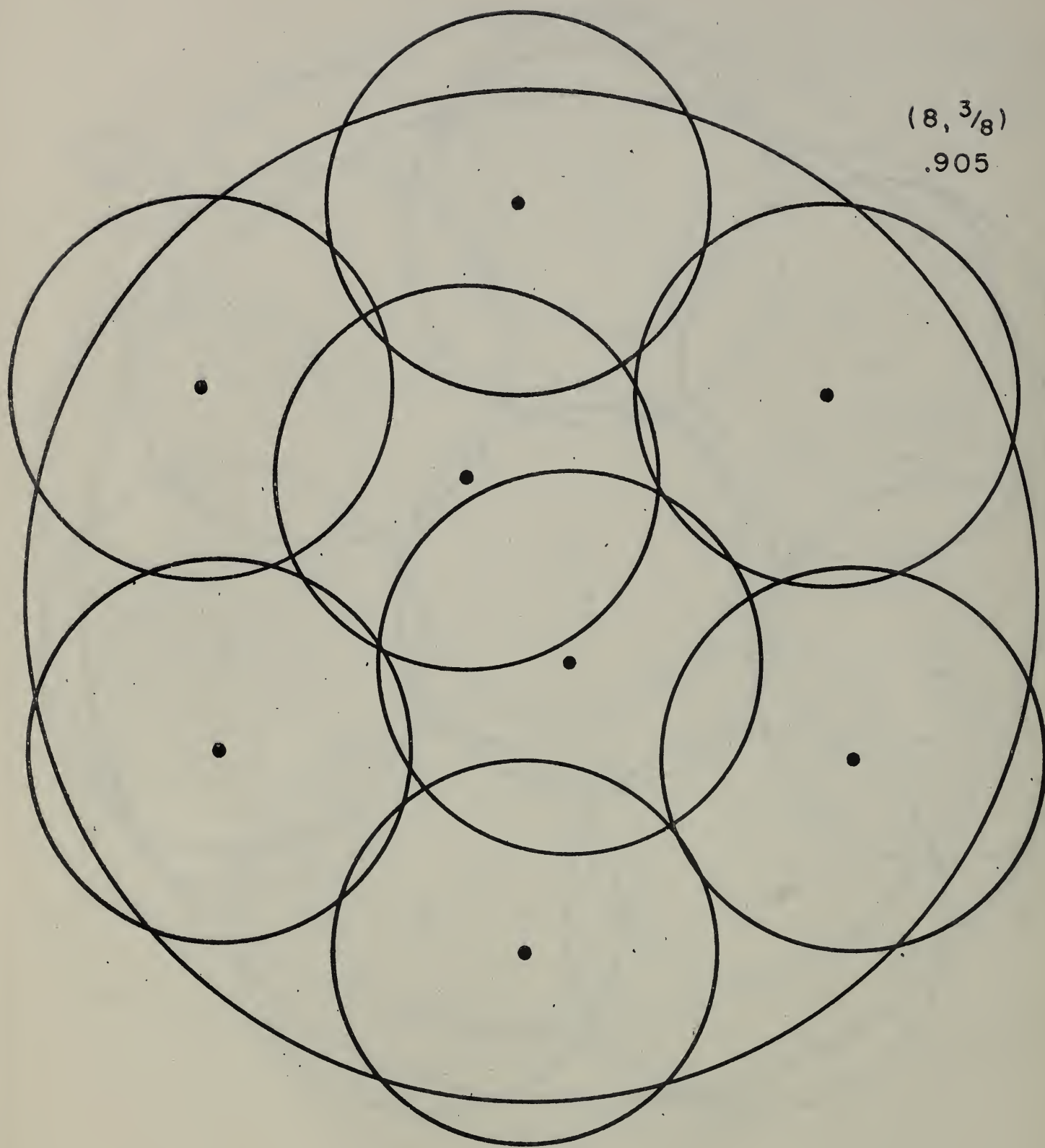


FIG. 60

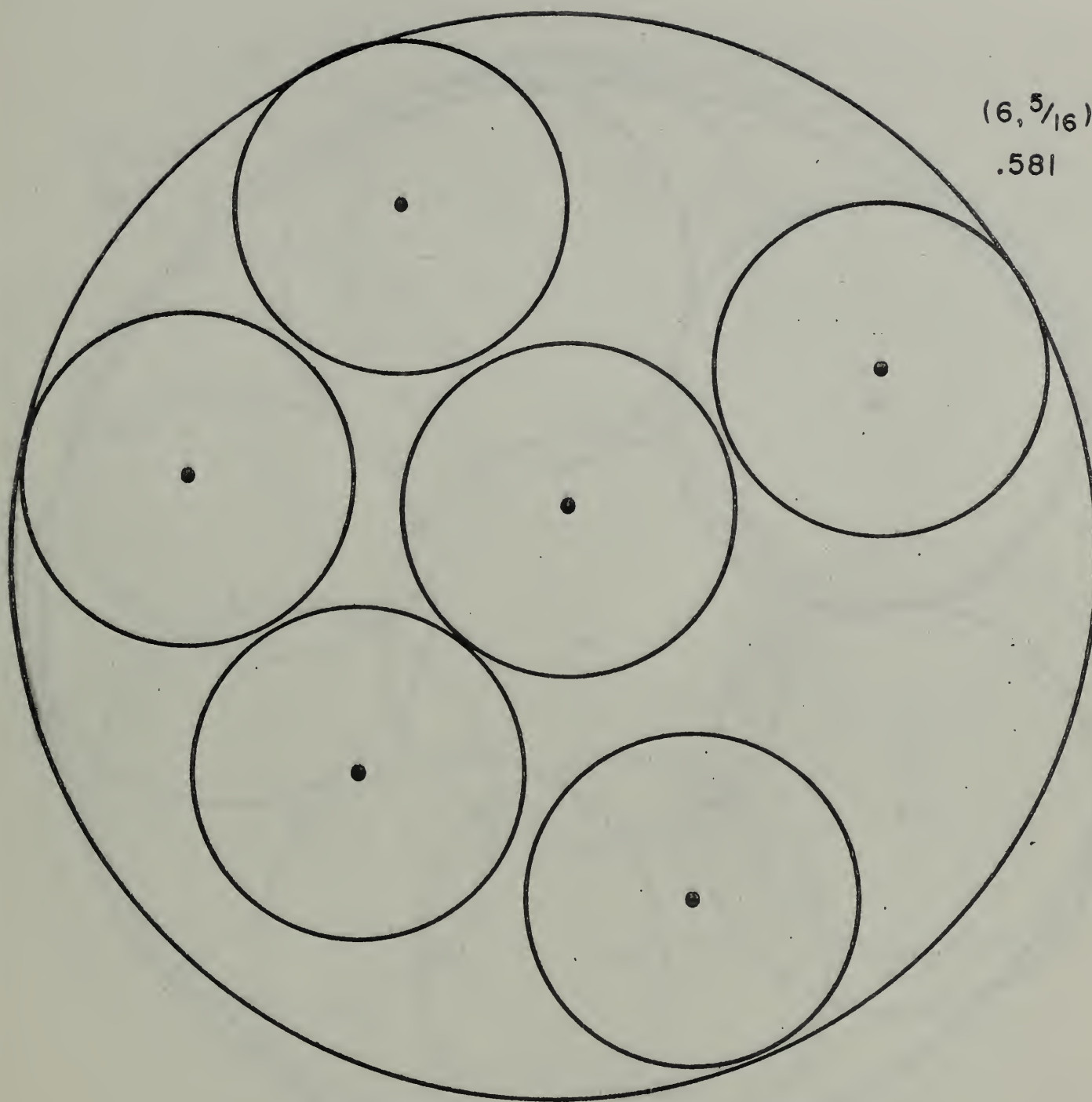
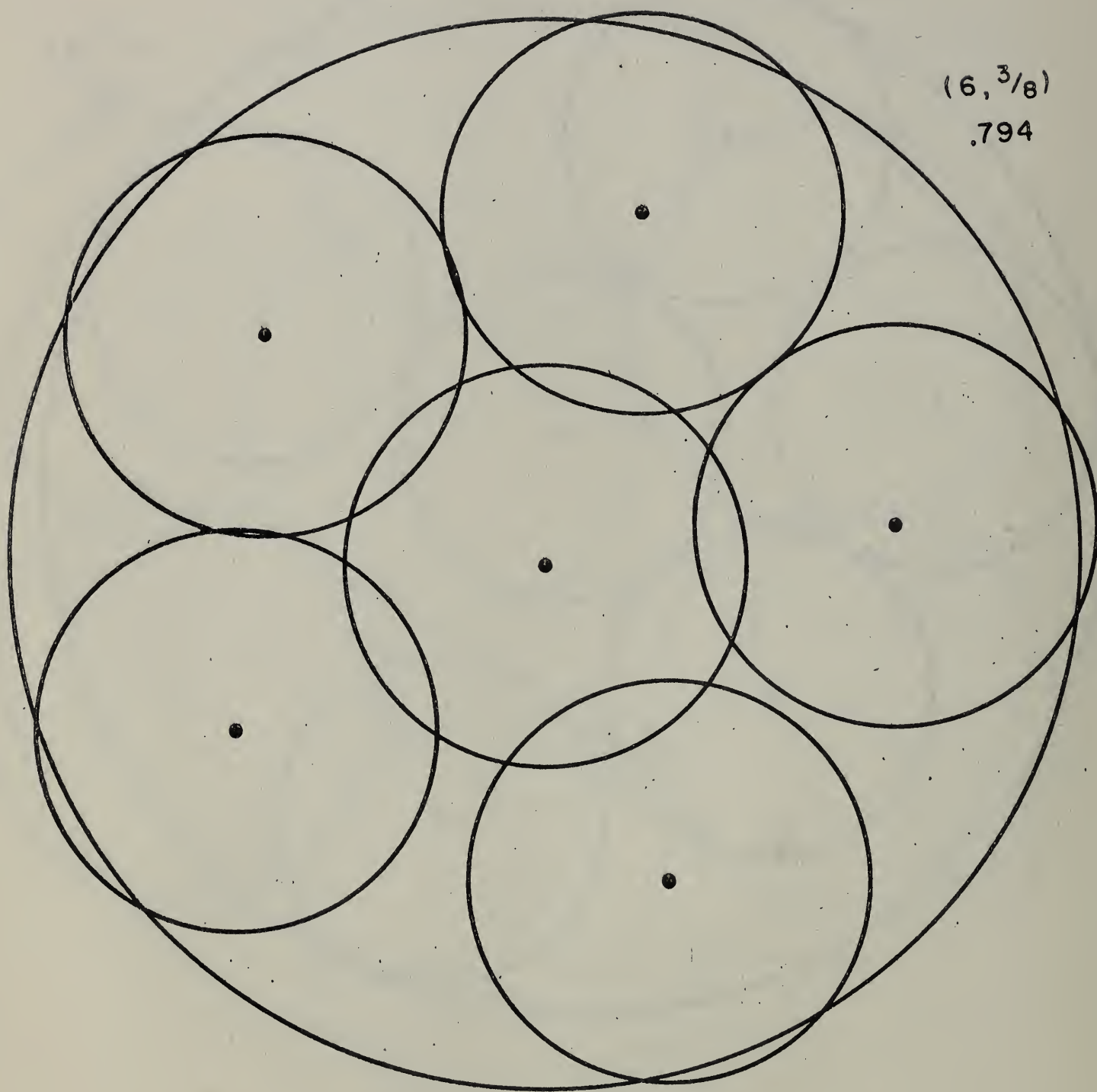


FIG. 61

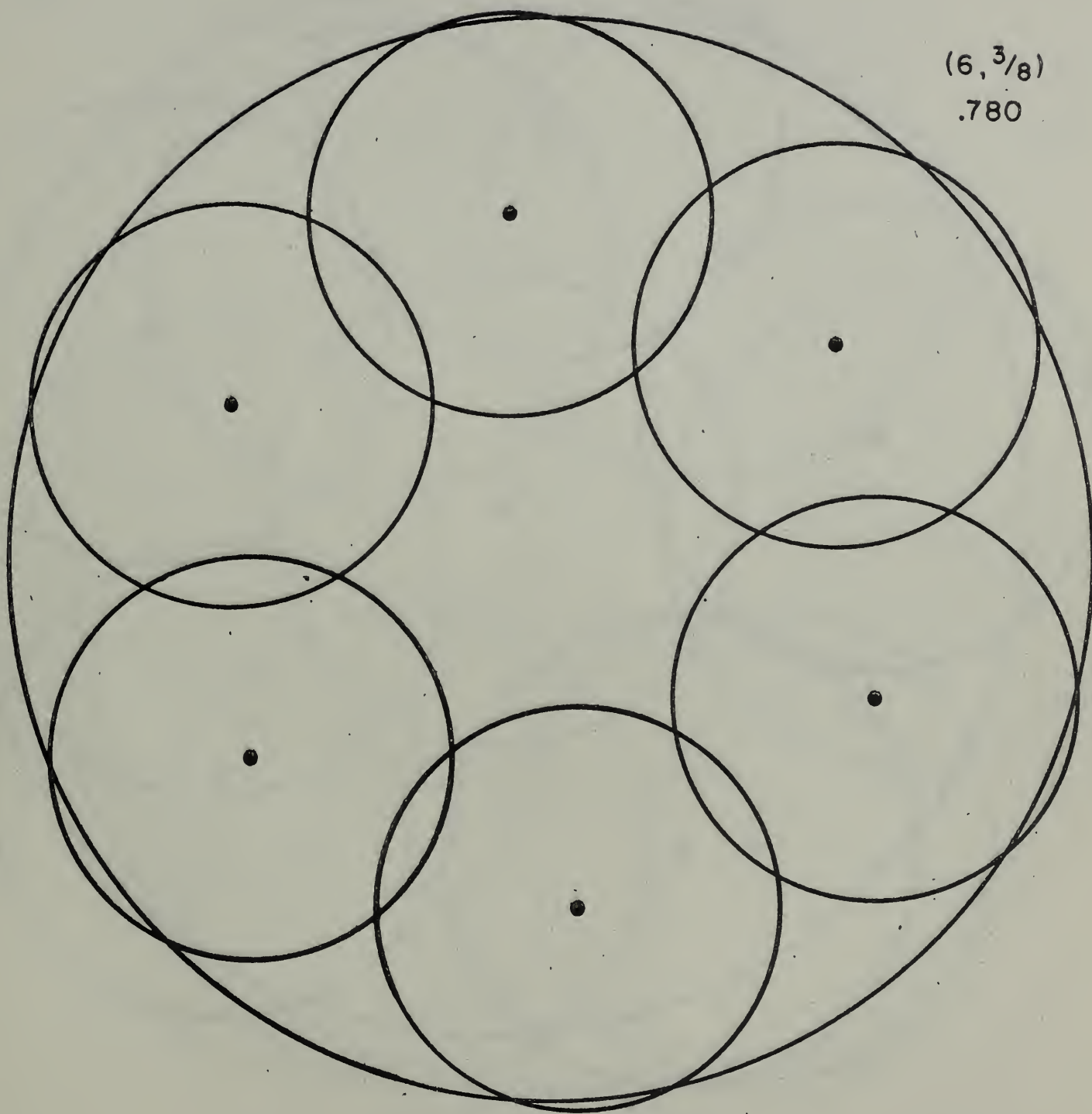


$(6, \frac{3}{8})$
.794

FIG. 62

$(6, \frac{3}{8})$

.780



FIG, 63

$(6, \frac{7}{16})$
.914

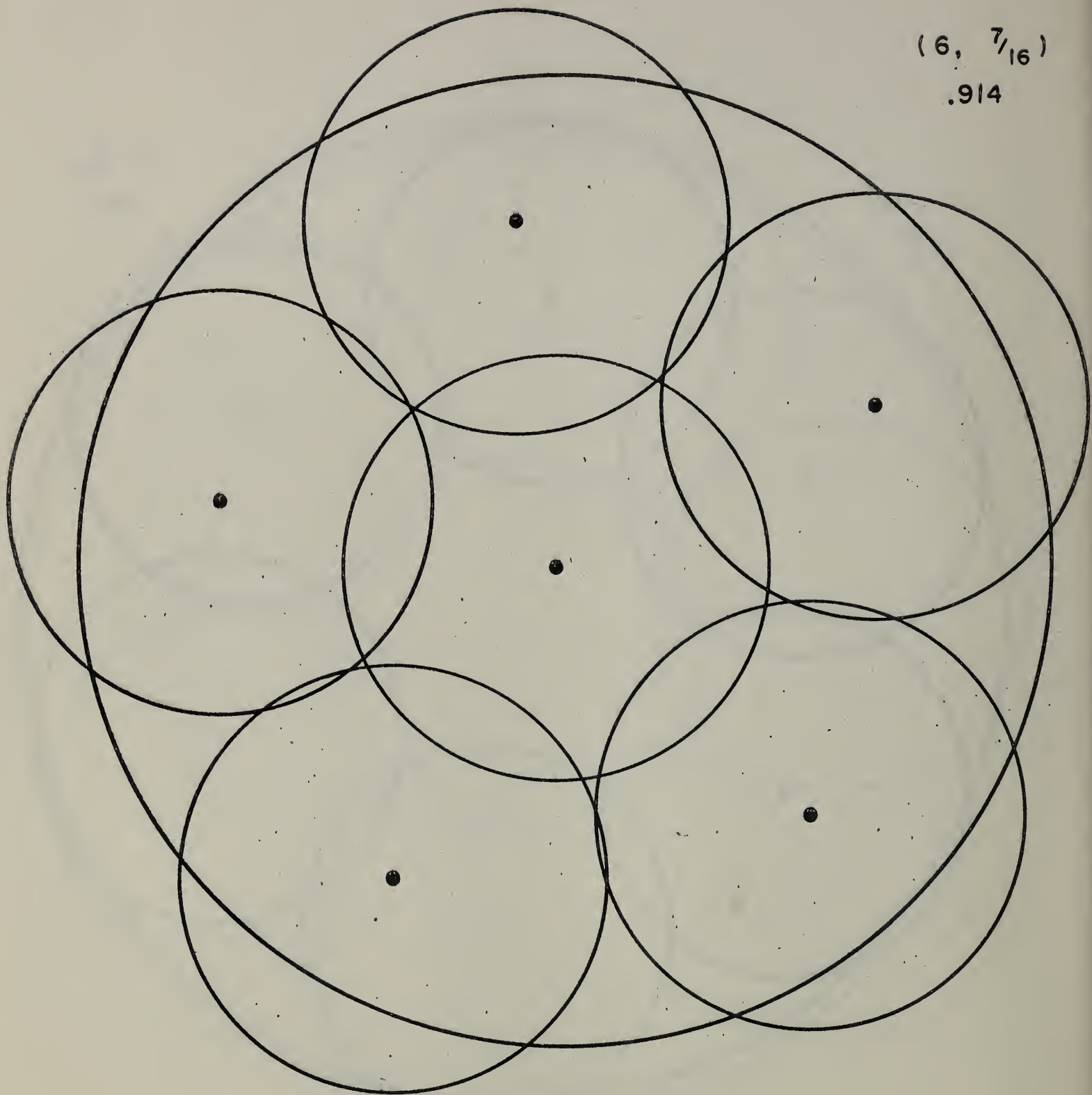


FIG. 64

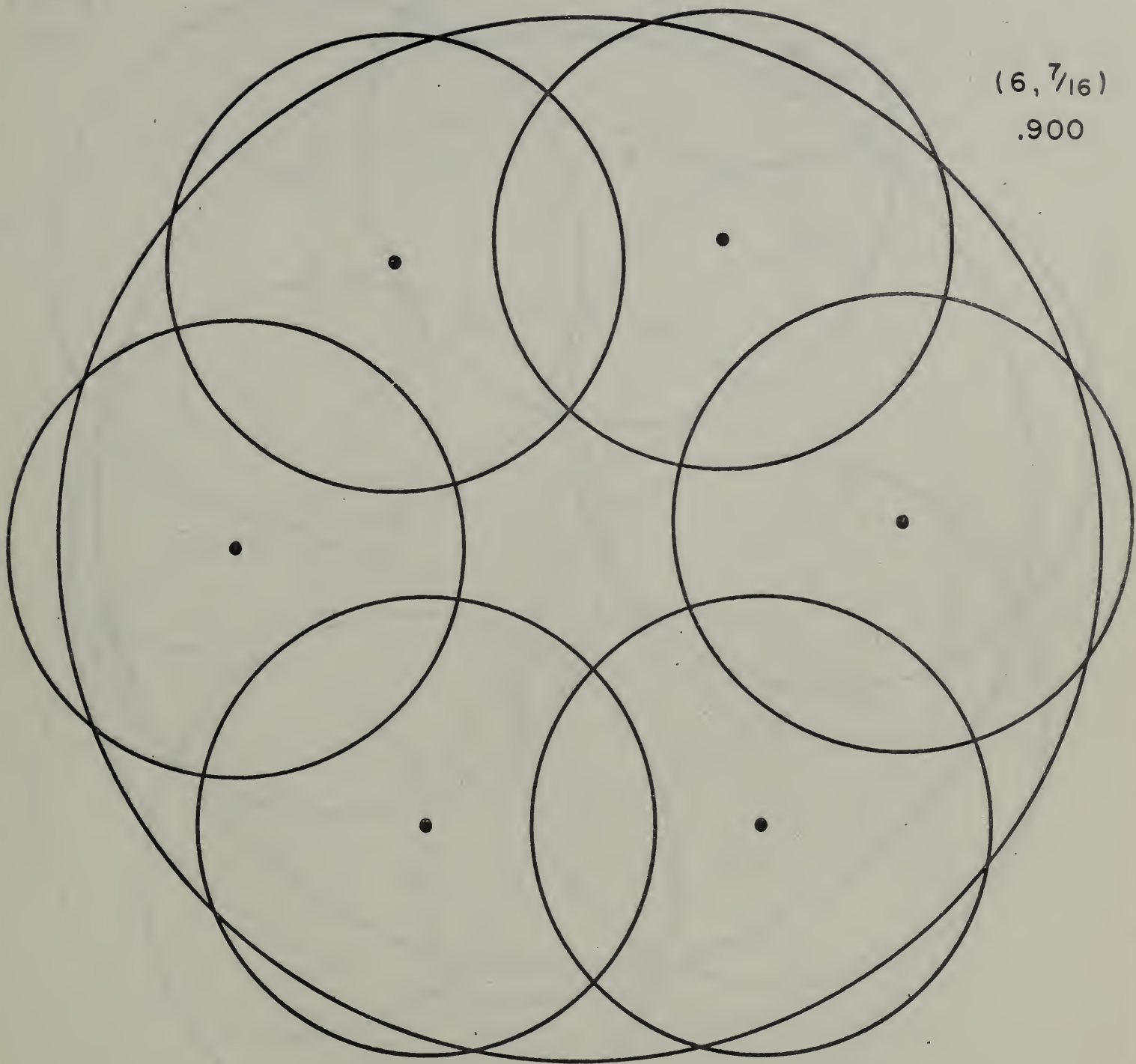


FIG. 65

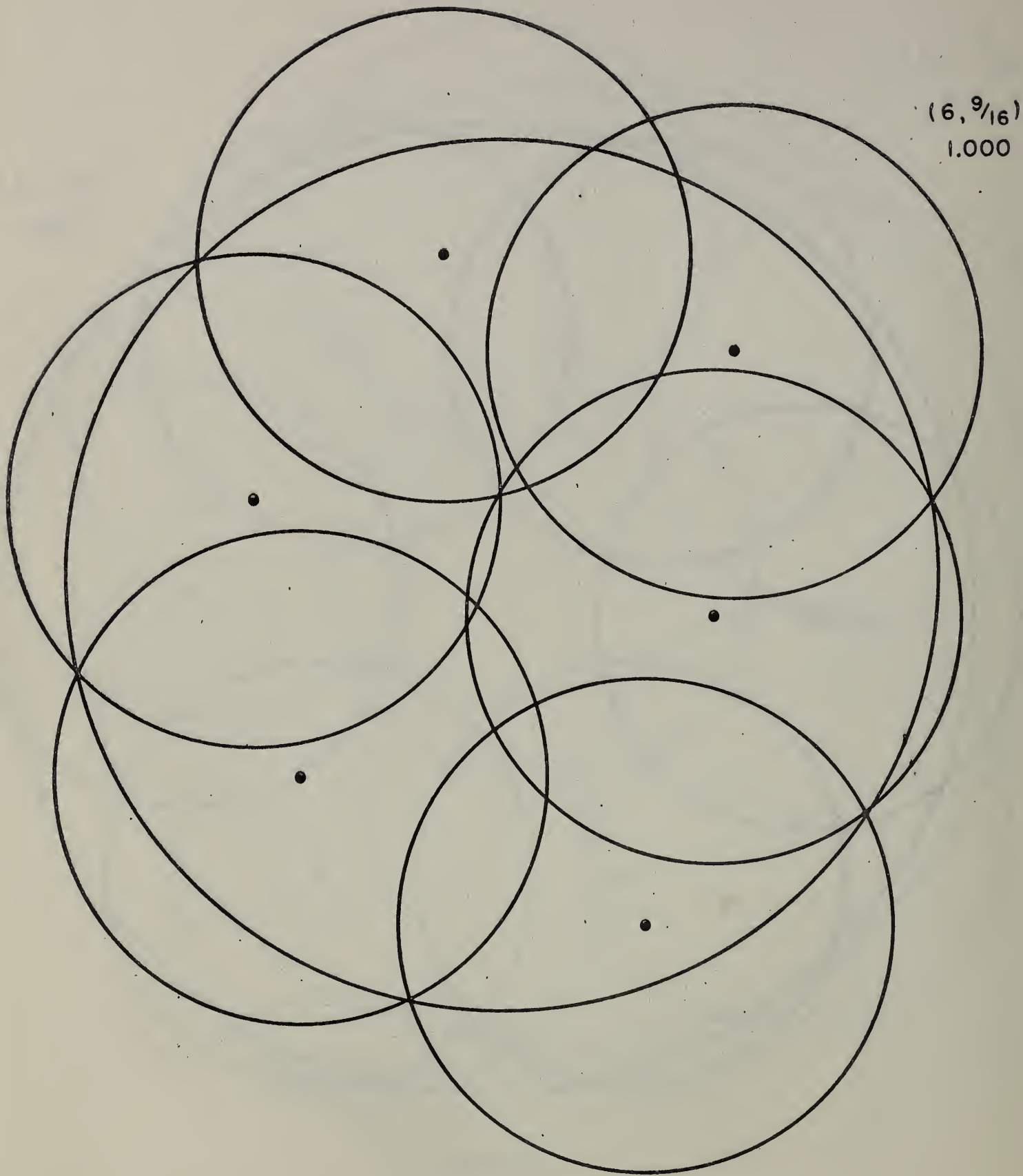


FIG. 66

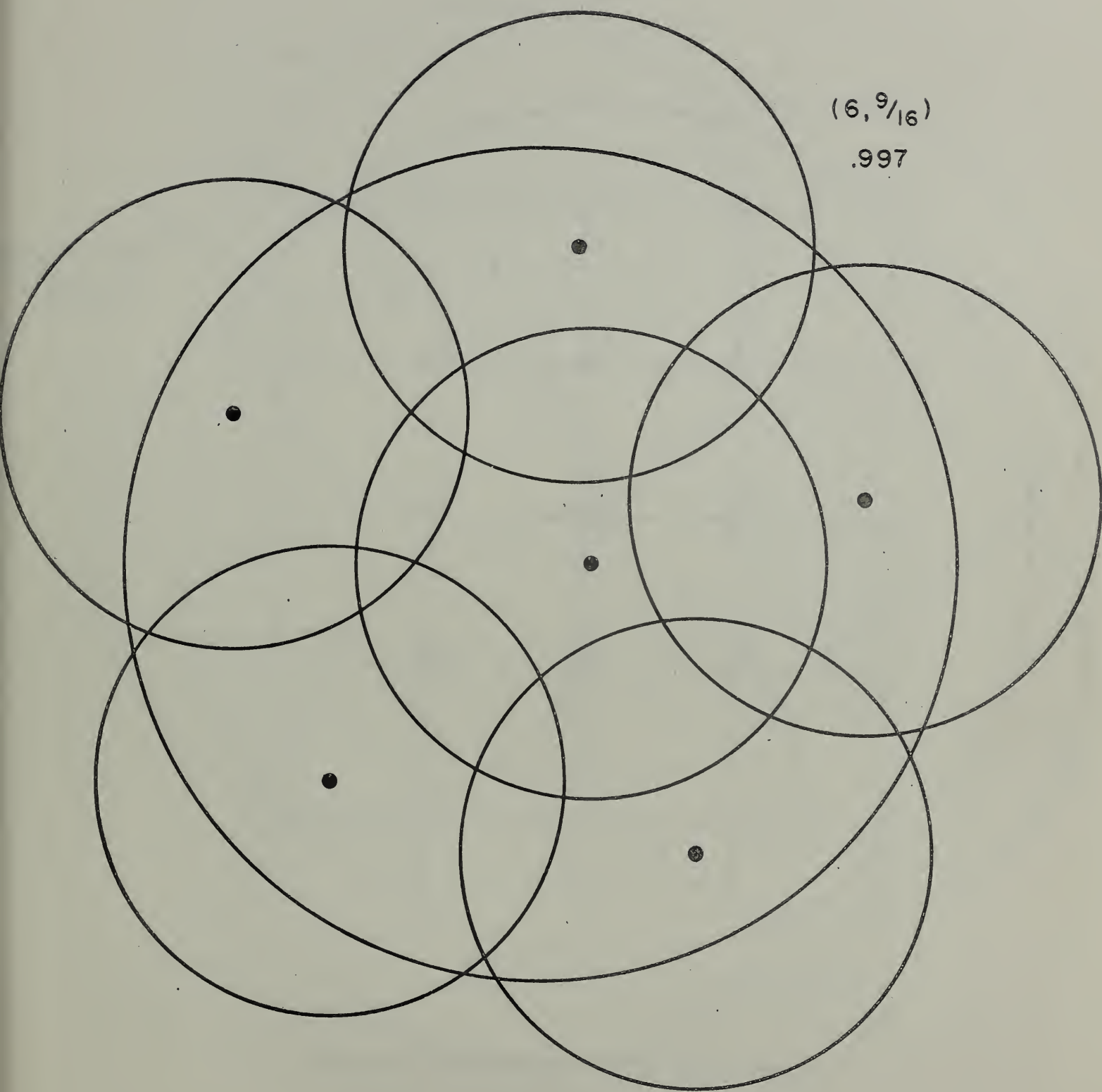


FIG. 67

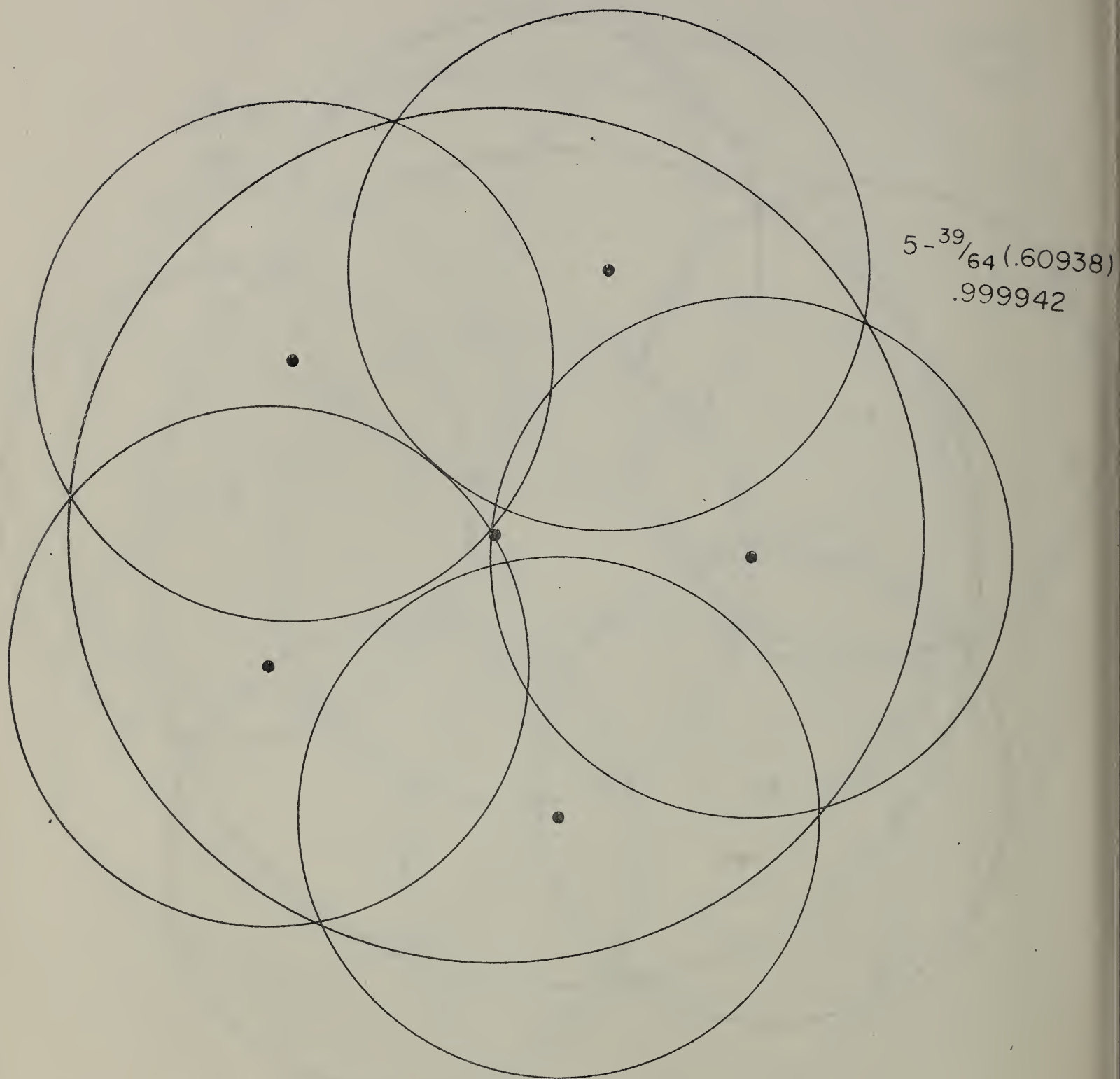


FIG. 68 COMPUTER CONFIGURATION FOR NEVILLE CASE

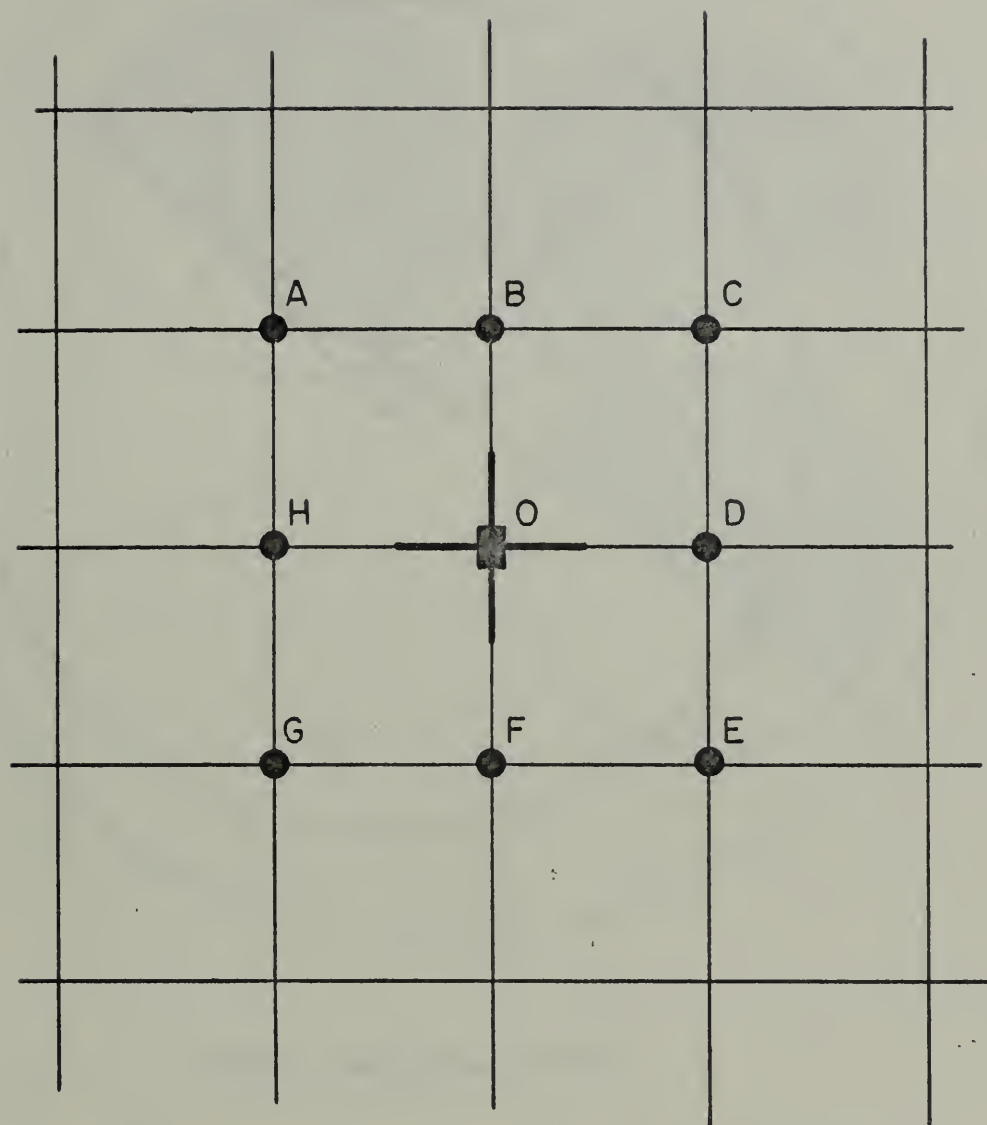


FIG. 69 NEIGHBORHOOD OF A MESH POINT

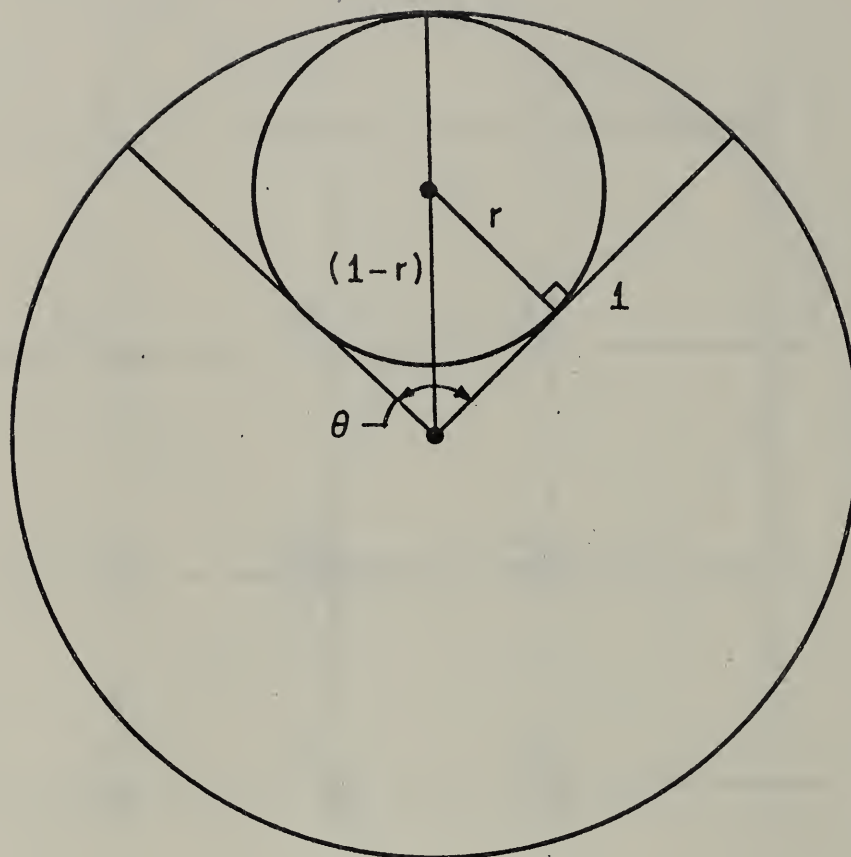


FIG.70 MAXIMUM PACKING

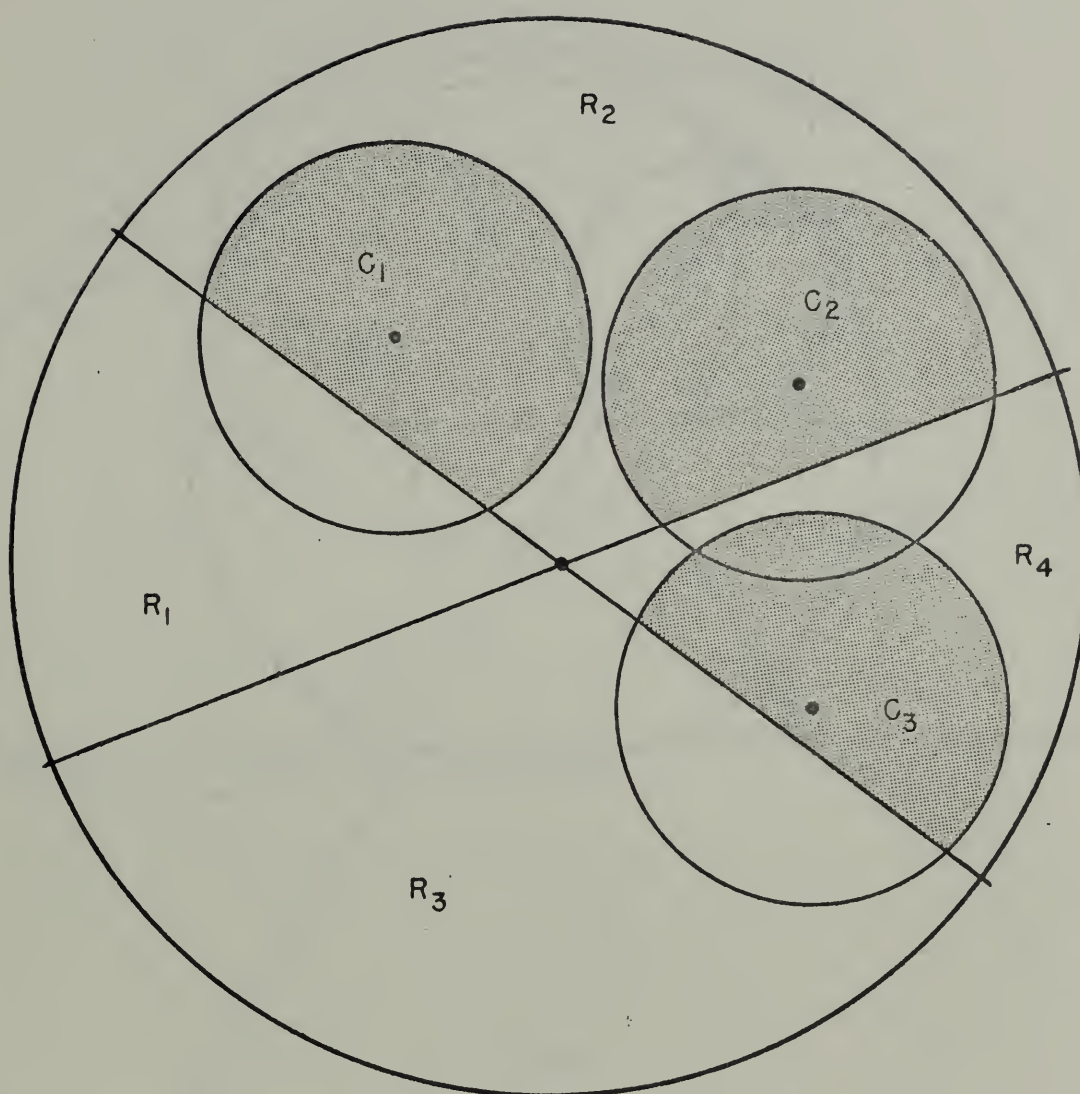


FIG. 71 WALL INTERRUPTED COVERAGE BY THREE DISCS.

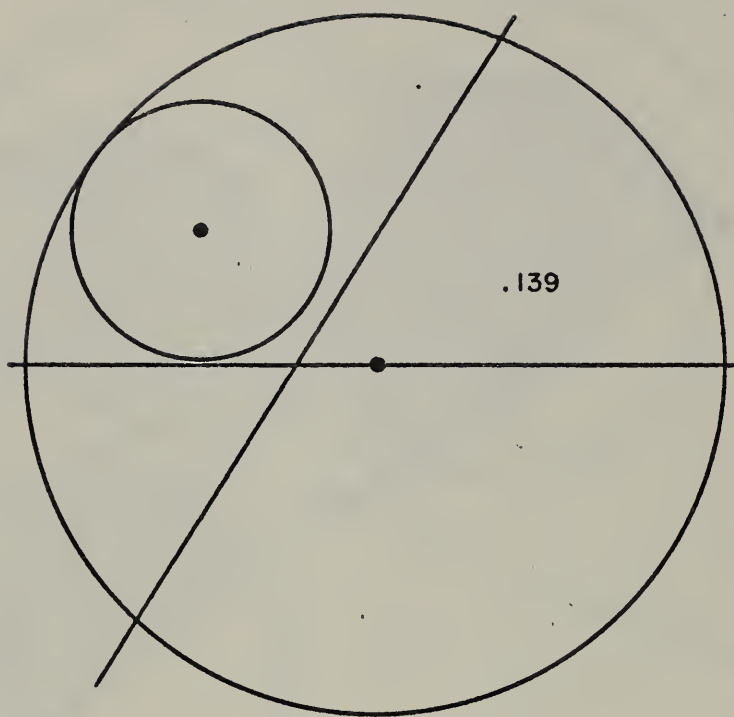


FIG. 72

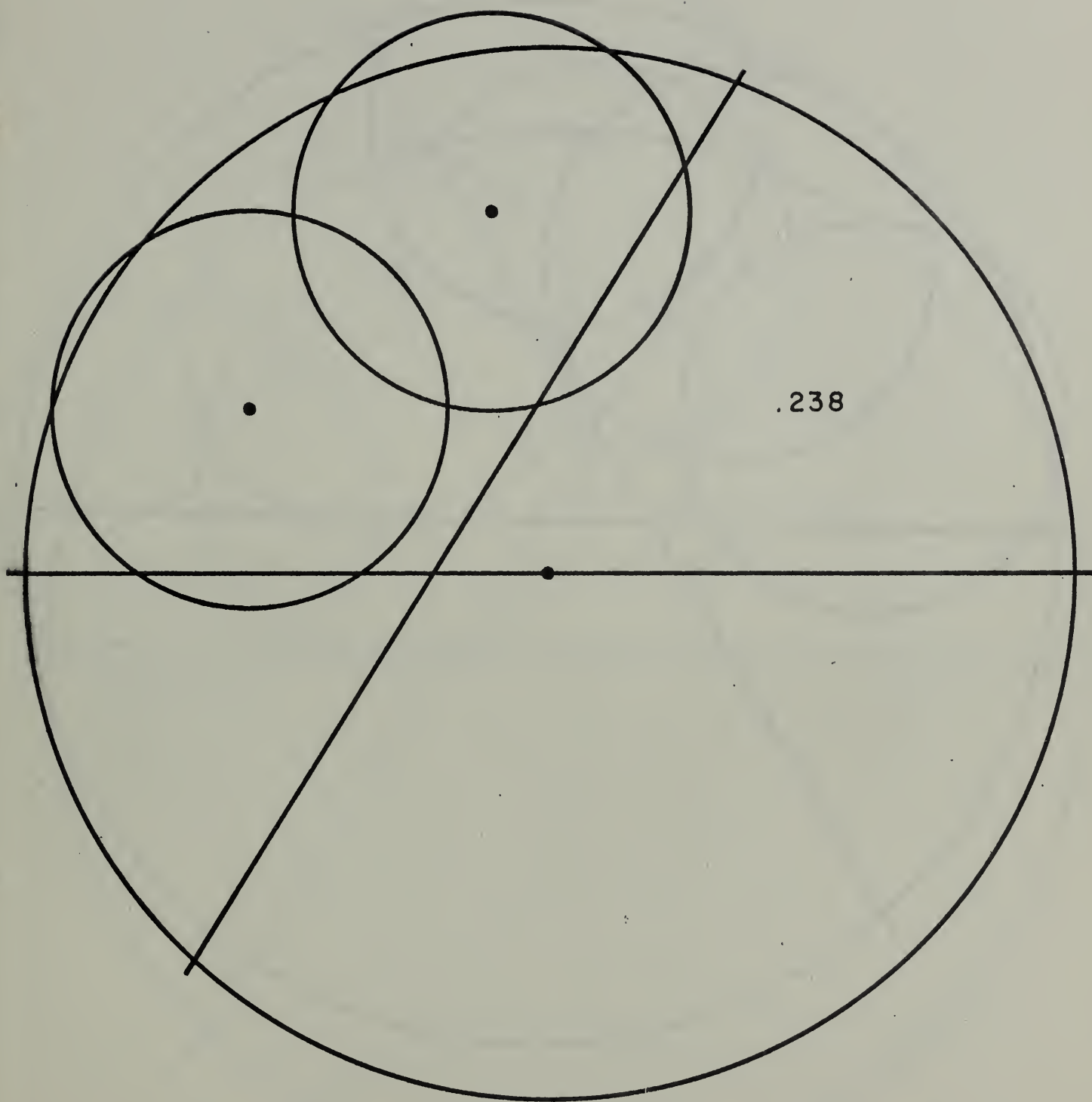


FIG.73

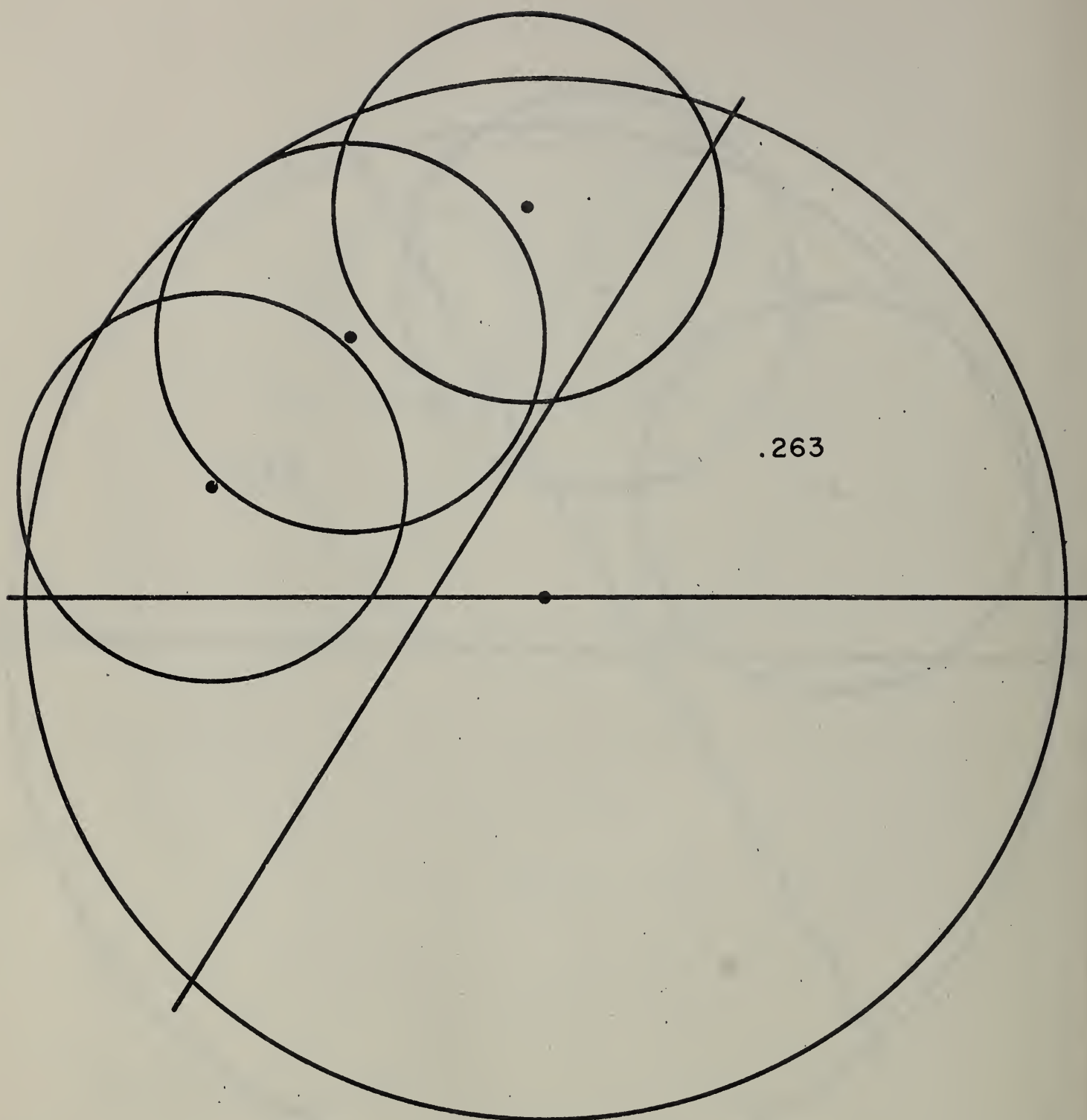


FIG. 74

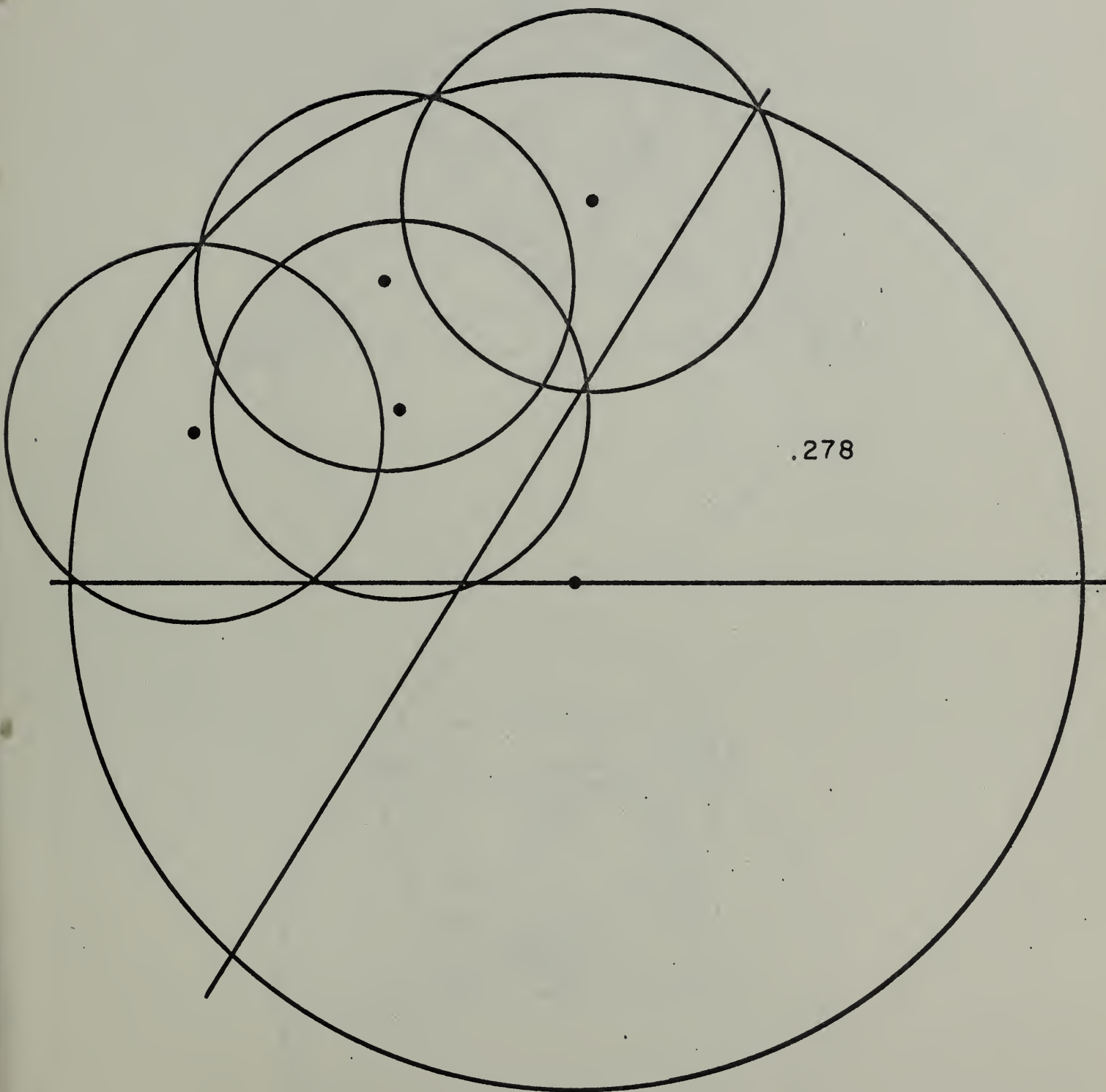


FIG. 75

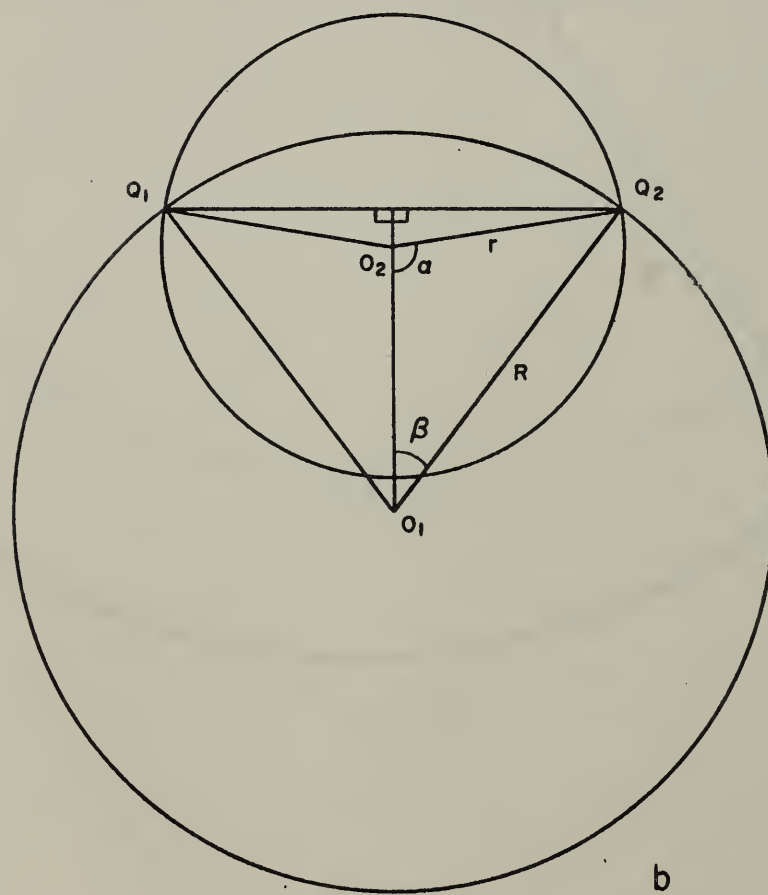
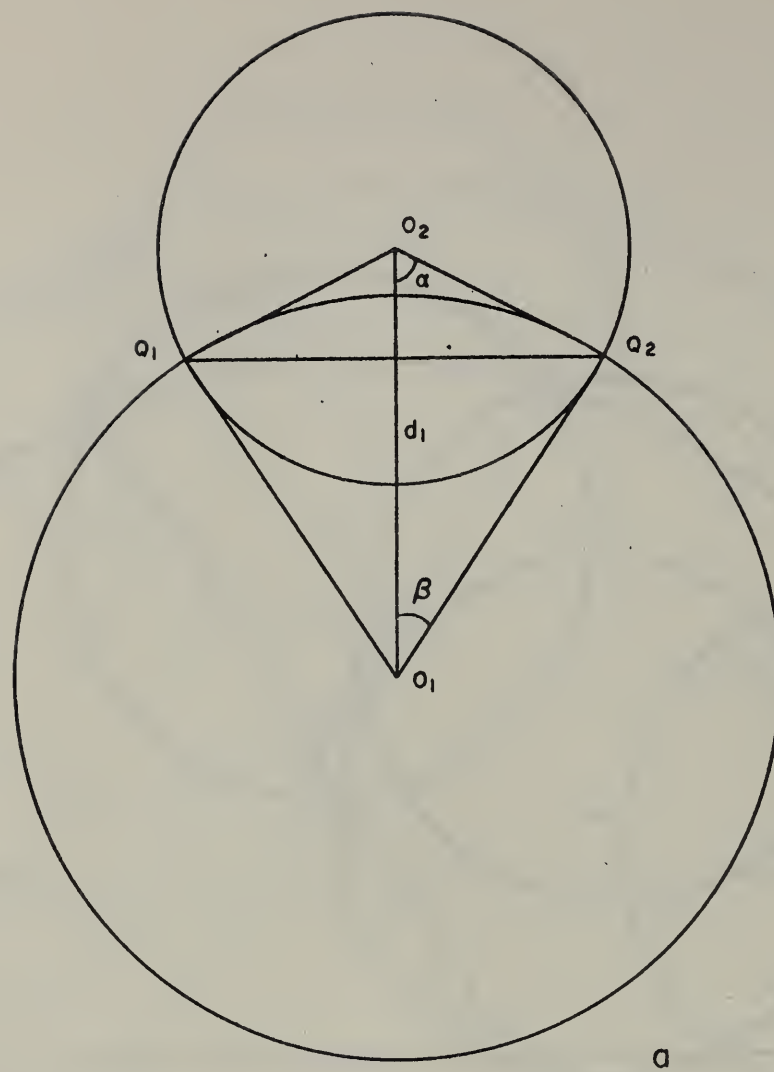
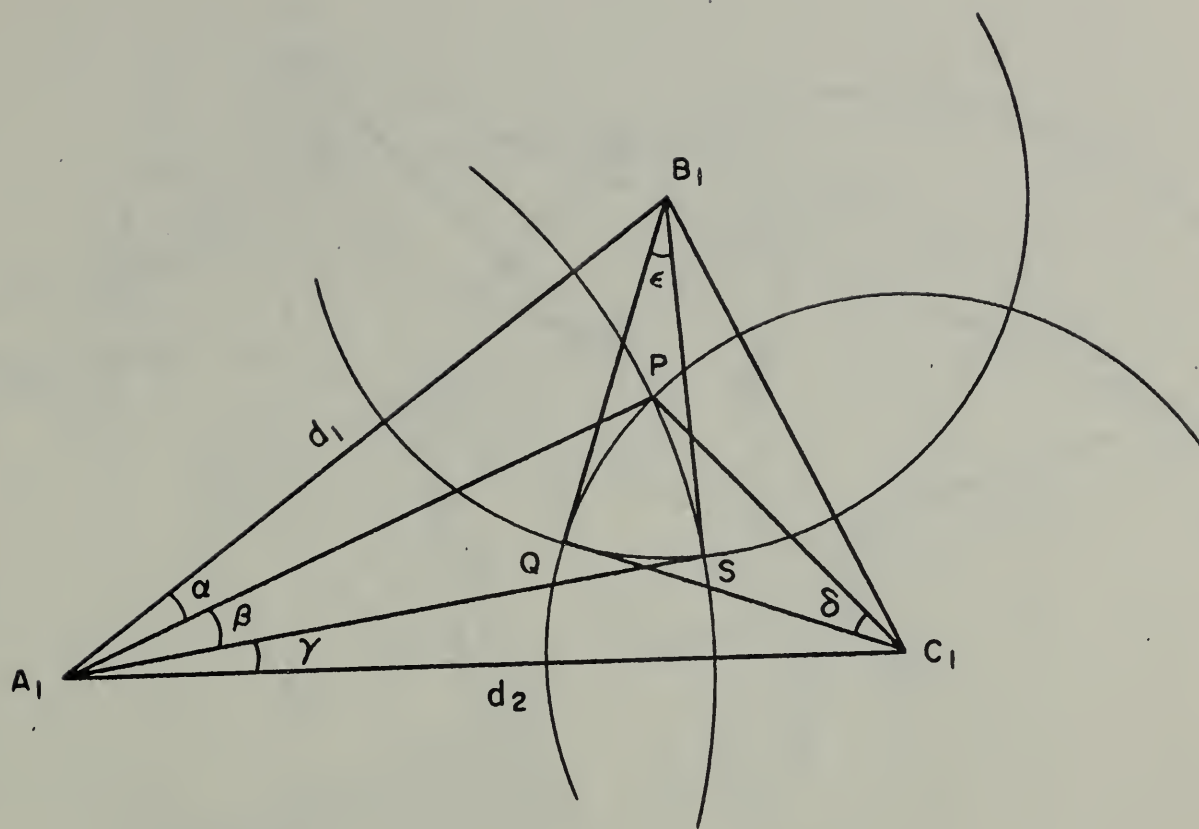
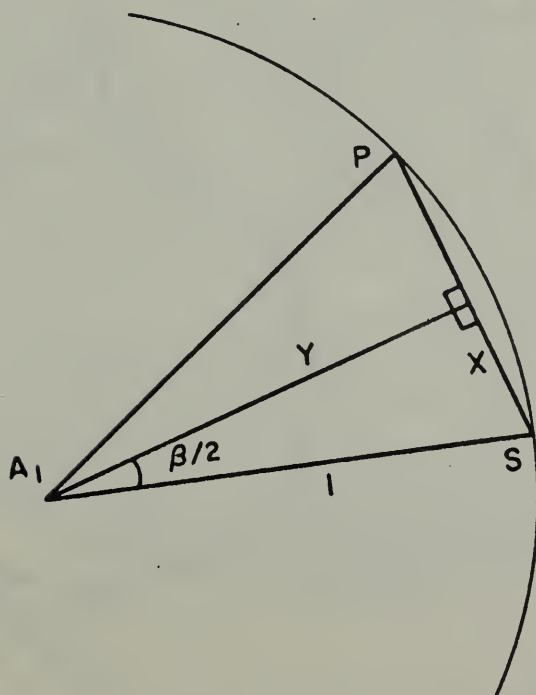


FIG. 76



a



b

FIG' 77

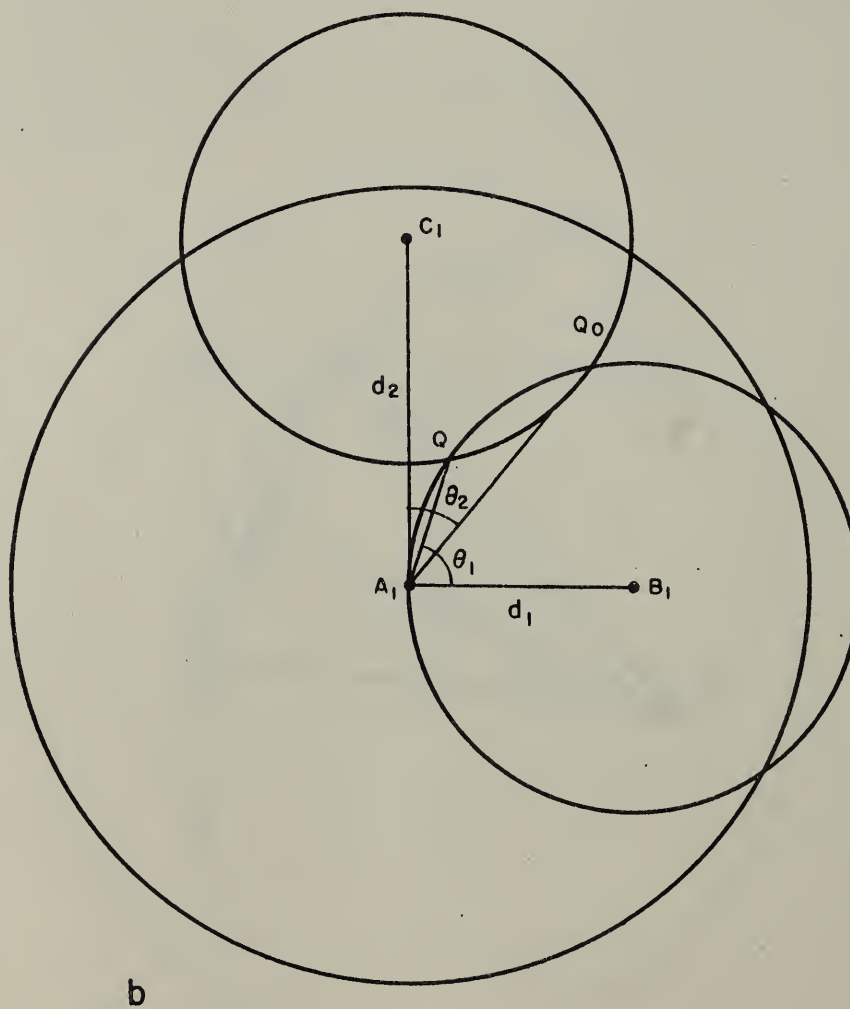
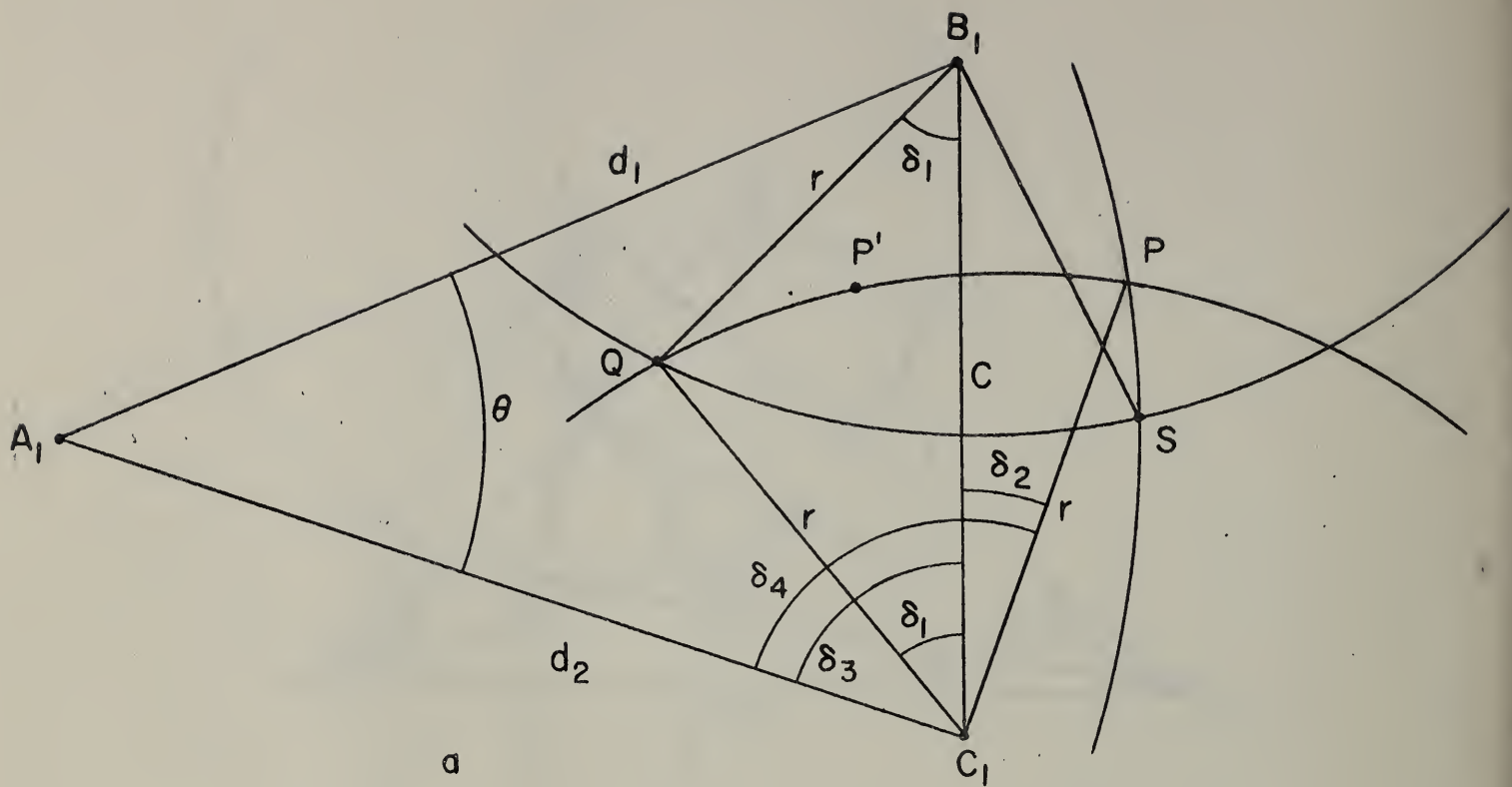


FIG. 78

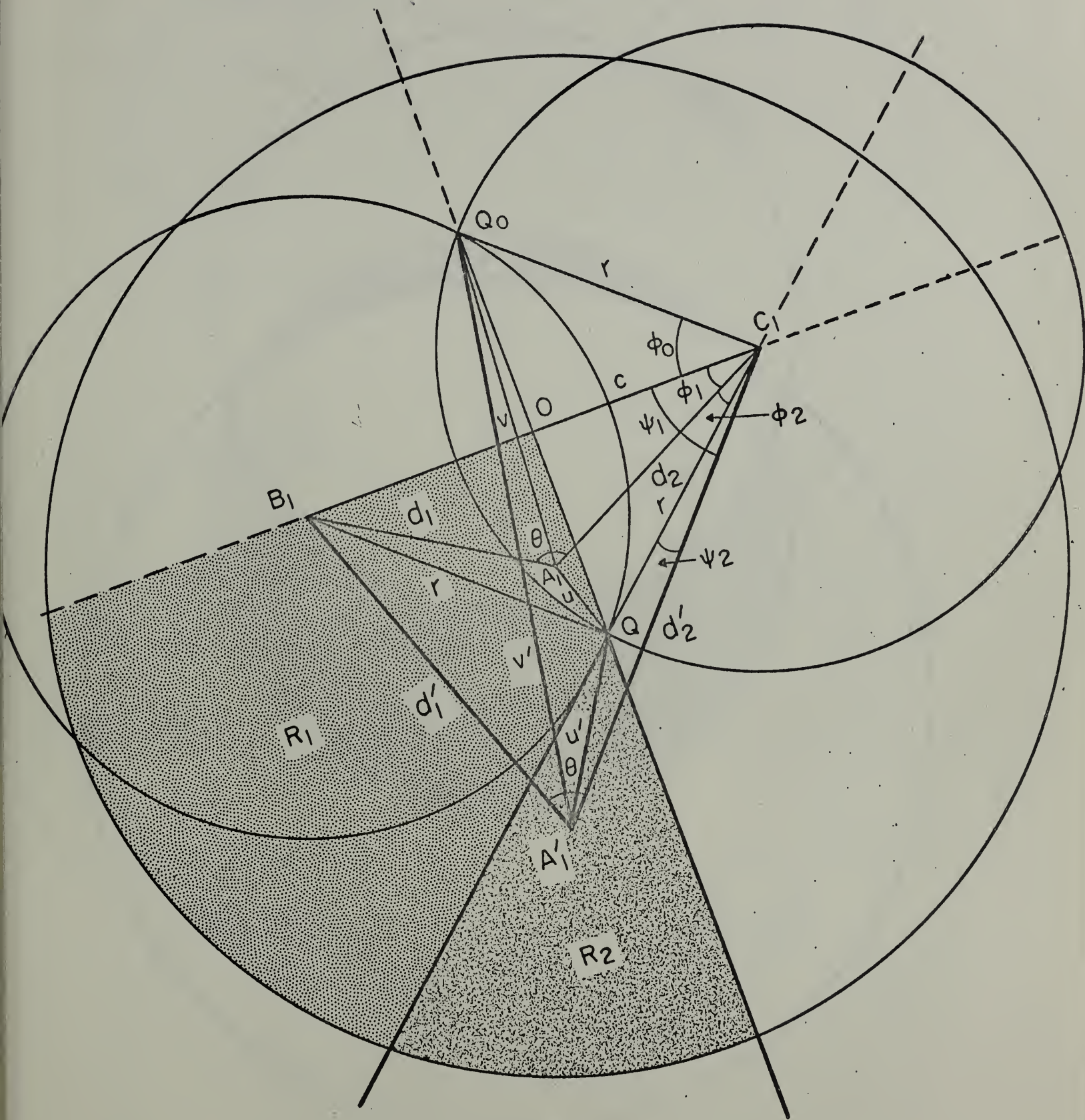


FIG. 79

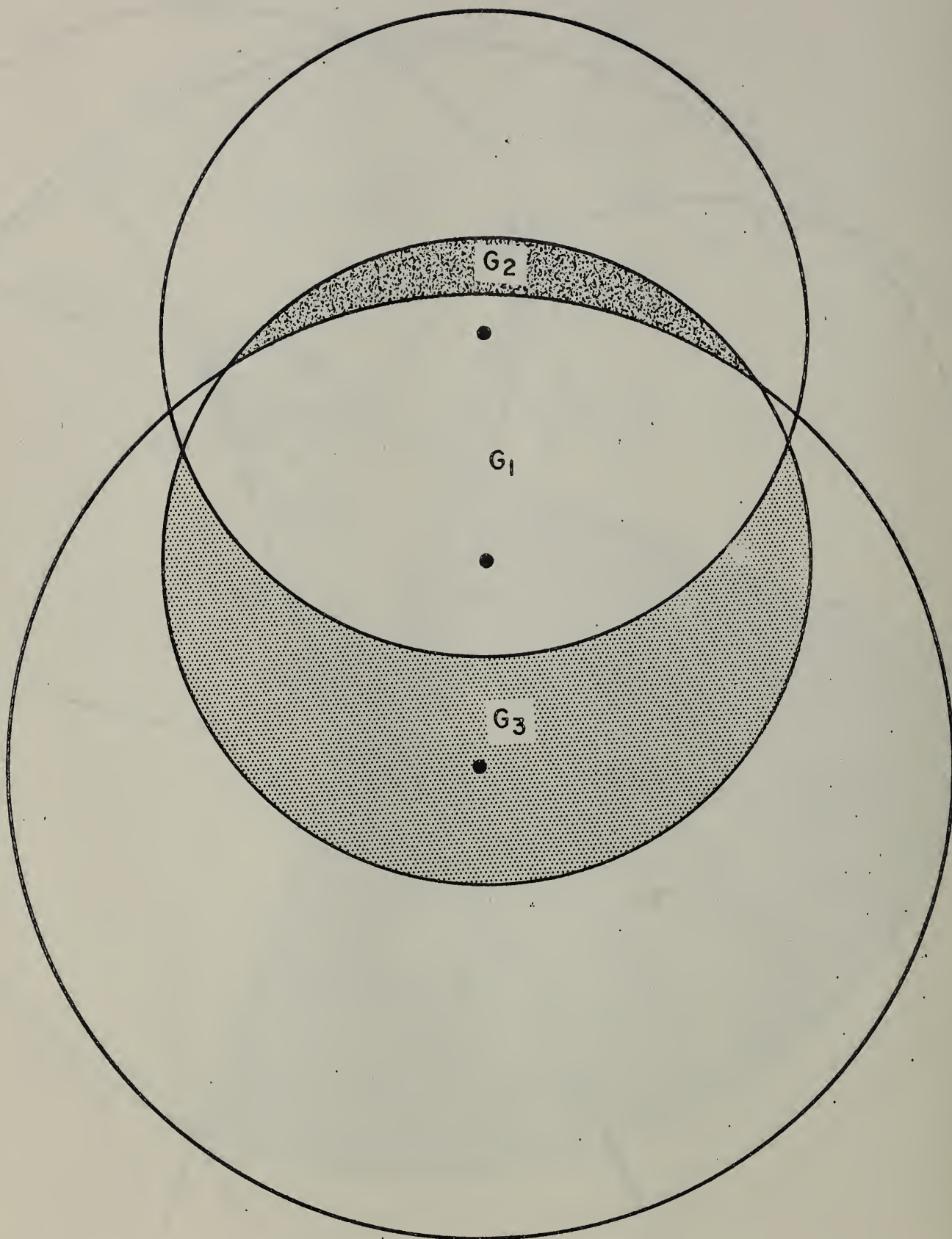
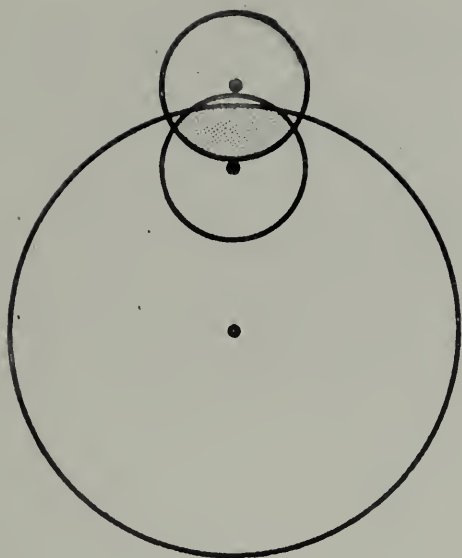
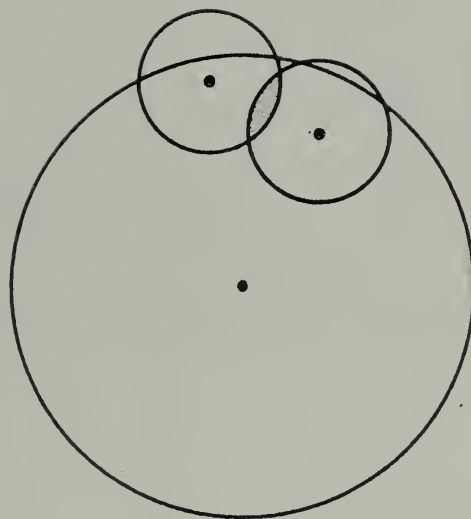


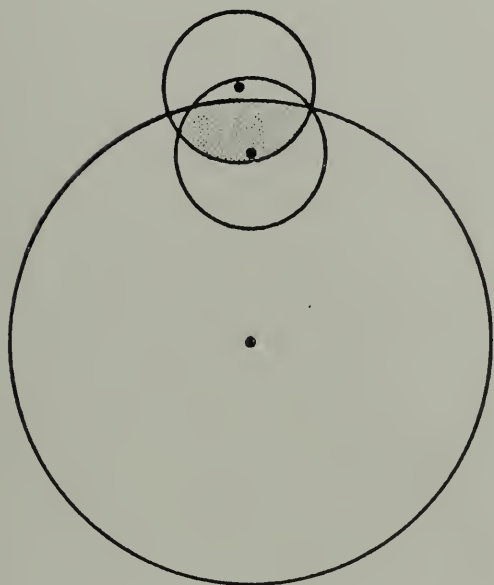
FIG. 80



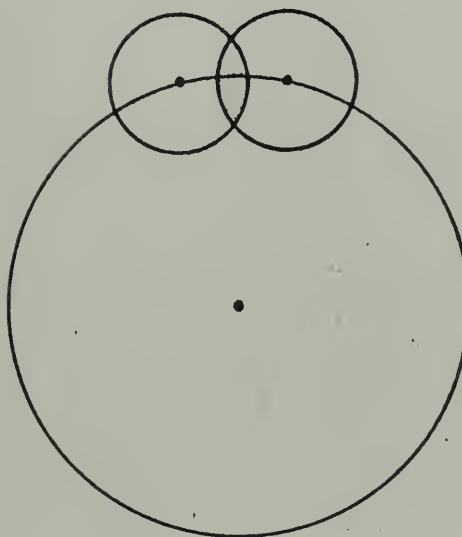
a



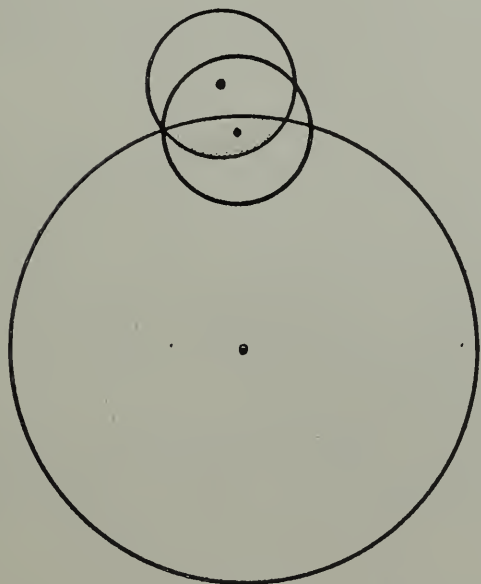
b



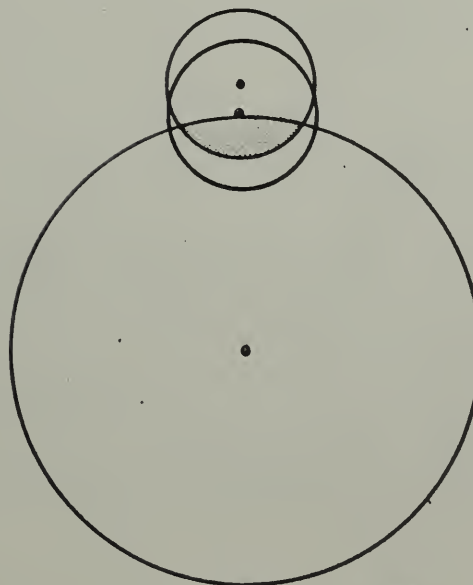
c



d



e



f

FIG. 81

U. S. DEPARTMENT OF COMMERCE

Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS

A. V. Astin, *Director*



THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D. C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics. **Radiation Physics.** X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Polymers. Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Microscopy and Diffraction. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics. **Instrumentation.** Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Molecular Kinetics. Mass Spectrometry.

Office of Weights and Measures.

BOULDER, COLO.

Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. Microwave Circuit Standards. Electronic Calibration Center.

